Search for Large, Compactified Extra Dimensions in the Diphoton Channel

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ABSTRACT

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The hierarchy problem has puzzled many physicists and represents a major shortcoming of the Standard Model. This problem can be phrased, “Why is gravity so weak compared to the other fundamental forces?” or, alternatively, “Why is the Planck mass scale so large compared to the mass scales of the other fundamental forces?” The ADD model of large extra dimensions posits that the Planck mass is actually of the same order of magnitude (TeV) as the weak mass scale, but it appears so large because gravity can propagate into extra dimensions of space while the other forces cannot, thus apparently rendering it weaker. One consequence of such a theory is that the graviton, a necessarily massless particle, would appear to have nonzero mass when observed in our 3D space. When the graviton decays into two photons, those photons can be measured and used to reconstruct the mass of the original graviton. In this analysis, the differential cross section for ADD and standard model diphoton production was computed and used to perform a Monte Carlo simulation that gave a distribution of diphoton masses. The efficiency of the CDF detector at measuring diphoton mass was calculated using a software package that simulates CDF’s response to particle events. That efficiency function was applied to the Monte Carlo simulation data to predict what the observable diphoton mass spectra should look like. After calculating the systematic error in this simulation, the CL_s technique was used to set lower limits on the two fundamental parameters of this model: the effective Planck mass M_s and number of extra dimensions n. At the 95% confidence level we place a lower limit on M_s of 1.76 TeV for $n = 2$, 1.98 TeV for $n = 3$, 1.66 TeV for $n = 4$, 1.50 TeV for $n = 5$, 1.40 TeV for $n = 6$, and 1.32 TeV for $n = 7$. These limits are not substantially more stringent than those set by Fermilab’s DØ collaboration two years ago. While our study had five times the luminosity, DØ’s study used a greater breadth of signals in their analysis, suggesting that following their procedure using our data will lead to limits on the ADD model parameters that are more stringent than those previously set.

INTRODUCTION

For decades, physicists have relied upon the Standard Model (SM), a theory intended to explain the fundamental particles and forces that have been observed to exist. For the most part, the Standard Model has been a runaway success, as it accurately describes three of the four fundamental forces. However, the Standard Model has thus far been unable to incorporate gravity. One of the reasons gravity is difficult to reconcile with the other forces is the fact that it is much weaker than the others. While the electroweak scale M_{EW} is of order 1 TeV, the Planck scale M_{Pl}, at which gravity becomes roughly equal in strength to the other forces, is of order 10^{16} TeV [1]. The ADD model — a theory [2] proposed by Arkani-Hamed, Dimopoulos and Dvali in 1998 — holds that M_{EW} is the only fundamental scale, and that the Planck mass is thus also of order TeV. This rescaling would make gravity roughly as potent as the electroweak force. The ADD model holds that we perceive gravity to be so weak (and M_{Pl} to be correspondingly large) because gravity is not confined to our intuitive three dimensions of space (the "brane"), but can also propagate into extra spatial dimensions (all together, the "bulk").

The idea, put simply, is this: imagine a spherical body with fixed mass m. We will represent its fixed mass by assigning it a fixed number of gravitational field lines (for example, 20). If gravity is restricted to propagating in two dimensions (analogous to the 3D brane we inhabit), then the field lines will be distributed equatorially around the source. If gravity is permitted to propagate in three dimensions, the field lines will be distributed around the entire surface area of the sphere, and will thus be spaced further apart, signifying a weakening of the field as observed from a test location.

If these extra spatial dimensions were infinite in size, or at least as large as the three that make up our brane, then we would observe a deviation from Newton's $1/r^2$ law of gravity. The

ADD model holds that these extra dimensions are compactified, i.e., of finite size R, such that for distances $r \gg R$ gravity appears to follow the $1/r^2$ force law. Only for distances $r \ll R$ would we see such deviations. From Newton's force law, it is possible to derive a gravitational analogue to Gauss's law:

$$M_{Pl}^2 \sim M_s^{n+2} R^2 \tag{1}$$

where M_{Pl} is the apparent Planck mass and M_s is the fundamental Planck mass. This equation gives a rough idea of the size of the extra dimensions. For M_s ~ TeV, $n = 1$ yields R greater than the size of the solar system. Since Newtonian gravitation is empirically confirmed on that size scale, n must be 2 or greater. $n = 2$ yields R in the millimeter range [1]. No experiments have been performed verifying that Newtonian gravity holds at such short distances, so non-Newtonian gravity is well within the realm of possibility.

The graviton is a particle that has not yet been discovered, but is theorized to exist. It is the theoretical carrier of the gravitational force, and since gravity propagates at the speed of light, the graviton must be a massless particle. However, one consequence of gravity's propagation into extra dimensions is that the graviton will appear to have mass when measured in the 3D brane. The relation between energy, momentum and invariant mass is:

$$E^2 = p^2c^2 + m^2c^4 \tag{2}$$

or, in natural units, where $c = 1$,

$$E^2 - p^2 = m^2 \tag{3}$$

In 3D space,

$$E^2 - p_x^2 - p_y^2 - p_z^2 = m_{3D}^2 \tag{4}$$

Because it exists in a 3+*n* – dimensional bulk, the graviton will have more than three components to its momentum. Without loss of generality, all the extra components can be collapsed into one term, yielding:

$$E^2 - p_x^2 - p_y^2 - p_z^2 - p_{extra}^2 = m_{G(bulk)}^2 = 0 \tag{5}$$

Substituting in equation (4),

$$m_{3D}^2 - p_{extra}^2 = 0 \tag{6}$$

or,

$$m_{3D}^2 = p_{extra}^2 \tag{7}$$

Thus, under the ADD model, the graviton will appear to have mass when measured in 3D space.

The graviton, if it exists, would be a very short-lived particle. To detect its presence, it would be necessary to observe its decay products. There are many possible channels for the graviton to decay. This study focuses on the decay of the graviton to two photons, henceforth known as a diphoton, or γγ. A paper published by Fermilab's DØ Collaboration in 2008 [3] also examines the electron-positron channel, and another from 2005 examines the dimuon channel. The other dilepton channel, $\tau^+\tau^-$, would be much more difficult to observe, owing to the high mass of those particles. While photons are massless particles, a diphoton produced from the decay of a massive particle encodes the invariant mass of that parent in the momenta and energies of the photons. For a massive graviton in the brane,

$$E_G^2 - p_G^2 = m_G^2 \tag{8}$$

If the two photons are labeled 1 and 2, conservation of energy and momentum give

$$(E_1 + E_2)^2 - (\overrightarrow{p_1} + \overrightarrow{p_2})^2 = m_G^2 \tag{9}$$

or, expanding,

$$(E_1^2 + E_2^2 + 2E_1E_2) - (p_1^2 + p_2^2 + 2\,\overrightarrow{p_1} \cdot \overrightarrow{p_2}) = m_G^2 \tag{10}$$

For an individual photon,

$$E^2 - p^2 = m_\gamma^2 = 0 \tag{11}$$

so the energy and momentum terms corresponding to the two individual photon masses can be subtracted, leaving:

$$2E_1E_2 - 2\,\overrightarrow{p_1}\cdot\overrightarrow{p_2} = m_G^2 = m_{\gamma\gamma}^2 \tag{12}$$

By measuring the energy and momentum of photon pairs that have decayed from a massive particle, it is possible to reconstruct the invariant mass of the parent particle.

MATERIALS AND METHODS

Nearly all the computerized analysis performed for this project was done using C++ and the C++-based ROOT software designed at CERN. ROOT is an extensive set of packages and libraries designed to process high-energy physics data and perform statistical analyses. Wherever software was used that was not written in some combination of C++ and ROOT, it will be indicated. All of the diphoton data used in this analysis comes from the Collider Detector at Fermilab (CDF), one of two large detectors on Fermilab's Tevatron accelerator.

Cheung and Landsberg [5] give the leading-order (LO) differential cross section of diphoton production in the ADD model:

$$\frac{d^3\sigma}{dM_{\gamma\gamma}\,dy\,dz} = K\left\{\sum_q \frac{1}{48\pi s\,M_{\gamma\gamma}} f_q(x_1)f_{\bar{q}}(x_2)\left[2e^4Q_q^4\frac{1+z^2}{1-z^2} + 2\pi e^2Q_q^2M_{\gamma\gamma}^4\eta(1+z^2) + \frac{\pi^2}{2}M_{\gamma\gamma}^8\eta^2(1-z^4)\right] + \frac{\pi}{256s}f_g(x_1)f_g(x_2)M_{\gamma\gamma}^7\eta^2(1+6z^2+z^4)\right\}$$

where (13)

$$\eta = \frac{\mathcal{F}}{M_s^4}$$

$$\mathcal{F} = \begin{cases} \log\left(\frac{M_s^2}{\hat{s}}\right), & \text{for } n = 2 \\ \frac{2}{n-2}, & \text{for } n \geq 2 \end{cases}$$

This equation depends on three variables: the invariant mass $m_{\gamma\gamma}$ of the diphoton, the rapidity y of the diphoton (in the collision reference frame) and $z = \cos\theta^*$, the cosine of theta-star in the Collins-Soper reference frame. The Collins-Soper frame is the rest frame of the diphoton, wherein the two photons have equal and opposite momenta; thus, each makes the same angle θ* with the beam direction [6], [7]. The free parameters in this cross section are the effective Planck mass, M_s, and the number of extra dimensions, n. The only place these parameters appear in the equation is in η, so it is easy to separate this cross section into terms that come from the Standard Model and terms that come from ADD gravitons. s is the square of the collision energy; that energy is 1.96 TeV for the Fermilab Tevatron. $f_q(x_i)$ are the parton distribution functions (PDFs) of partons q participating in the collision (up, anti-up, down, anti-down, gluon), and $x_{1,2} = \frac{M_{\gamma\gamma}}{\sqrt{s}}e^{\pm y}$ determine how much of the hadron momentum is contained within the parton in question. $\hat{s}$ is equivalent to $m_{\gamma\gamma}^2$. PDFs were computed using data from the CTEQ Collaboration [8], with software written in Fortran. K is a rescaling constant for which the authors used 1.3, but it is irrelevant to this analysis because only the shape of the distribution, not its absolute normalization, determined our results.

Using this differential cross section for diphoton production, several Monte Carlo simulations were performed. For set values of the two parameters, 10^{10} diphoton events were generated using either the SM terms only or using SM and ADD terms, and various cuts were made to select useful diphoton events. Diphotons were only selected if they had a mass greater than 30 GeV and less than 1 TeV, and each photon had a rapidity with absolute value less

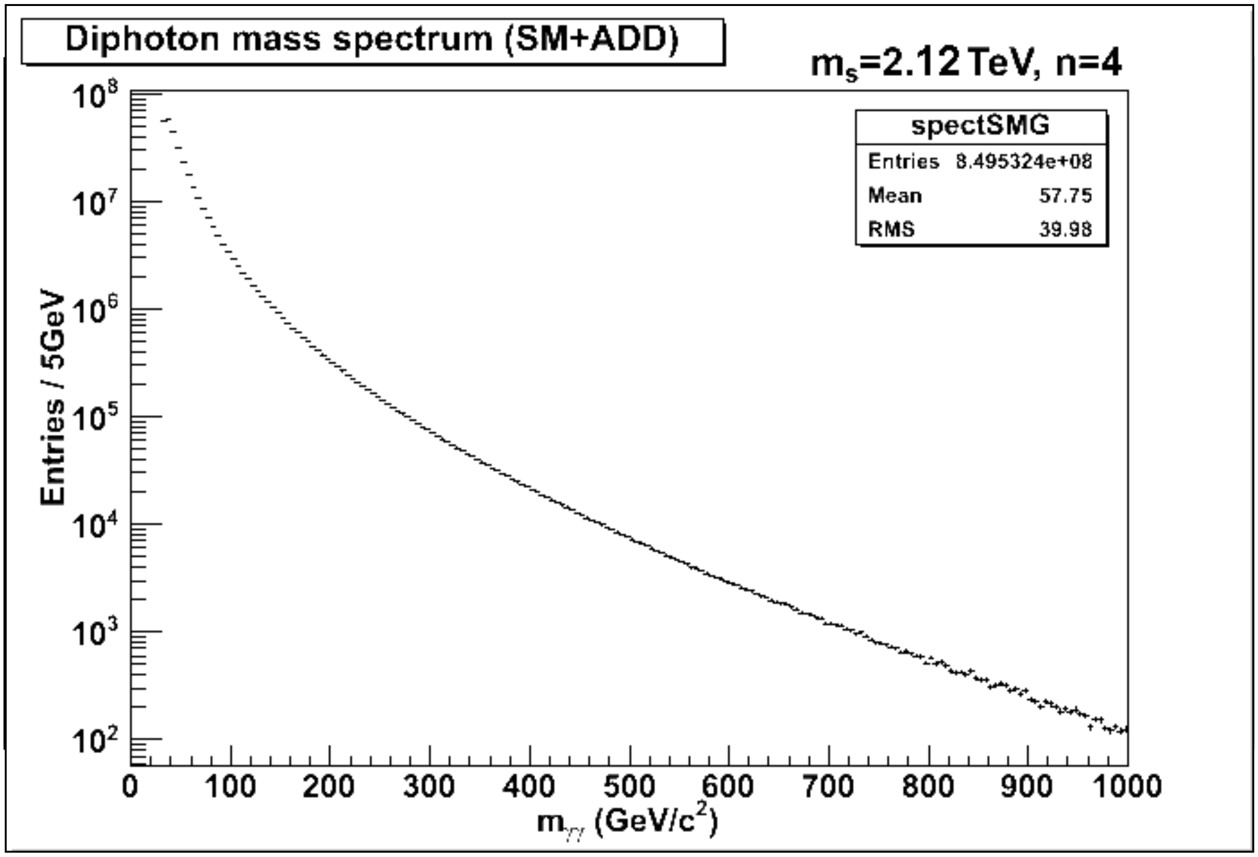

Figure 1: Example result of Monte Carlo simulation of diphoton events, after kinematic cuts.

than one (i.e., would register in the central calorimeter of the CDF detector) and a transverse momentum $p_T > 15$ GeV. These cuts resulted in a selection rate of approximately 8.5%, or 850 million events per simulation (Figure 1). The results of the SM-only simulation were very consistent with the LO results of prior simulations performed using DIPHOX, a next-to-leading-order (NLO) diphoton simulator, thus confirming the validity of the differential cross section (Figure 2).

The data from the Monte Carlo simulations were then weighted using the photon ID efficiency of the CDF detector. We took older diphoton Monte Carlo data and fed it to a simulation of CDF in order to gauge the detector's response to a well-described series of events. Efficiency is a non-constant function of $m_{\gamma\gamma}$. To account for any non-uniform response along the length of the detector, efficiency was calculated separately for five regions: $|z|$ between 0 and 0.2,

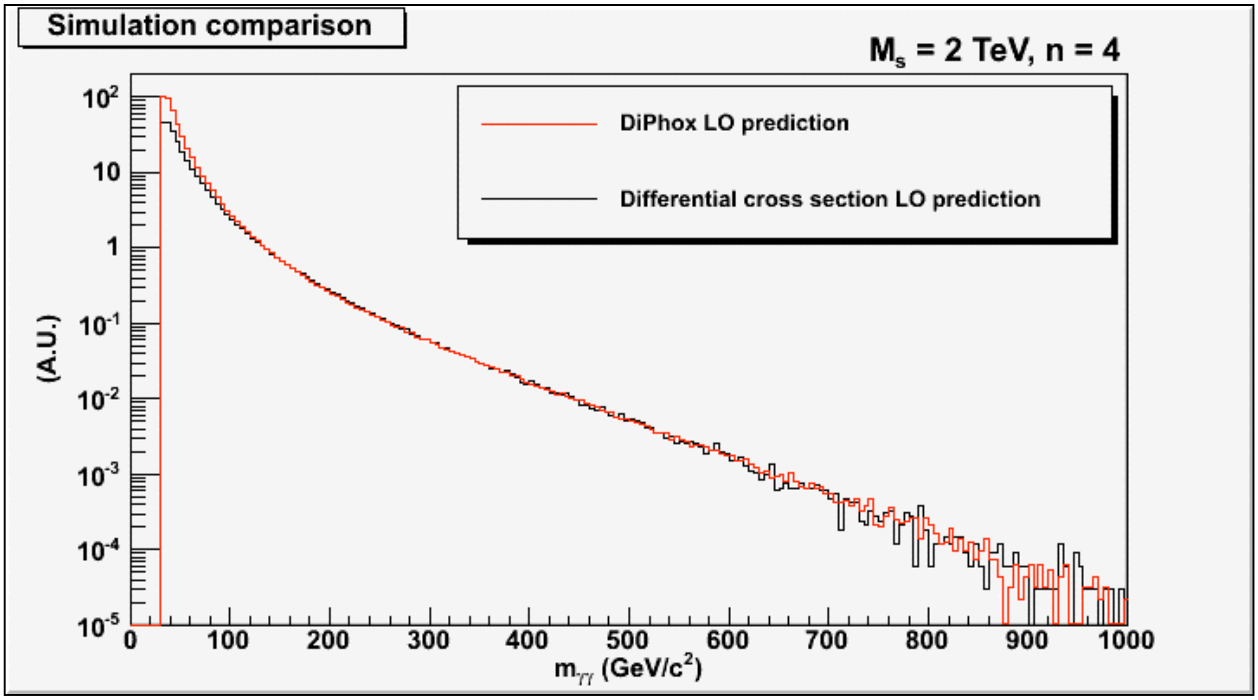

Figure 2: Comparison between Monte Carlo simulation using differential corss section and simulation from DIPHOX. Plots are normalized over the range 300-1000 GeV. Discrepancy at low mass is thought to be a result of DIPHOX using slightly different selection cuts.

between 0.2 and 0.4, and so on (Figure 3). Monte Carlo-simulated events were sorted into these same five bins and were multiplied by the corresponding efficiency function for that bin. This rescaling gave predictions for the diphoton mass spectra that one might expect to see from CDF (Figure 4). A comparison plot shows that using z-weighted efficiency gives nearly identical results to using the overall efficiency function. For greater accuracy, z-weighted efficiency was used when predicting expected CDF results.

During its Run II, the CDF detector saw a luminosity of 5.4 fb^{-1} and measured 1 471 diphotons heavier than 100 GeV that passed the kinematic and selection cuts mentioned above. As part of a prior analysis of the related Randall-Sundrum model, DIPHOX was used to fit these data to a prediction curve of SM diphotons and jets faking photons. In order to evaluate the agreement of the data with the SM prediction and the SM+ADD predictions, it was necessary to normalize the SM+ADD predictions to the scale of the DIPHOX fit. Our Monte Carlo data was

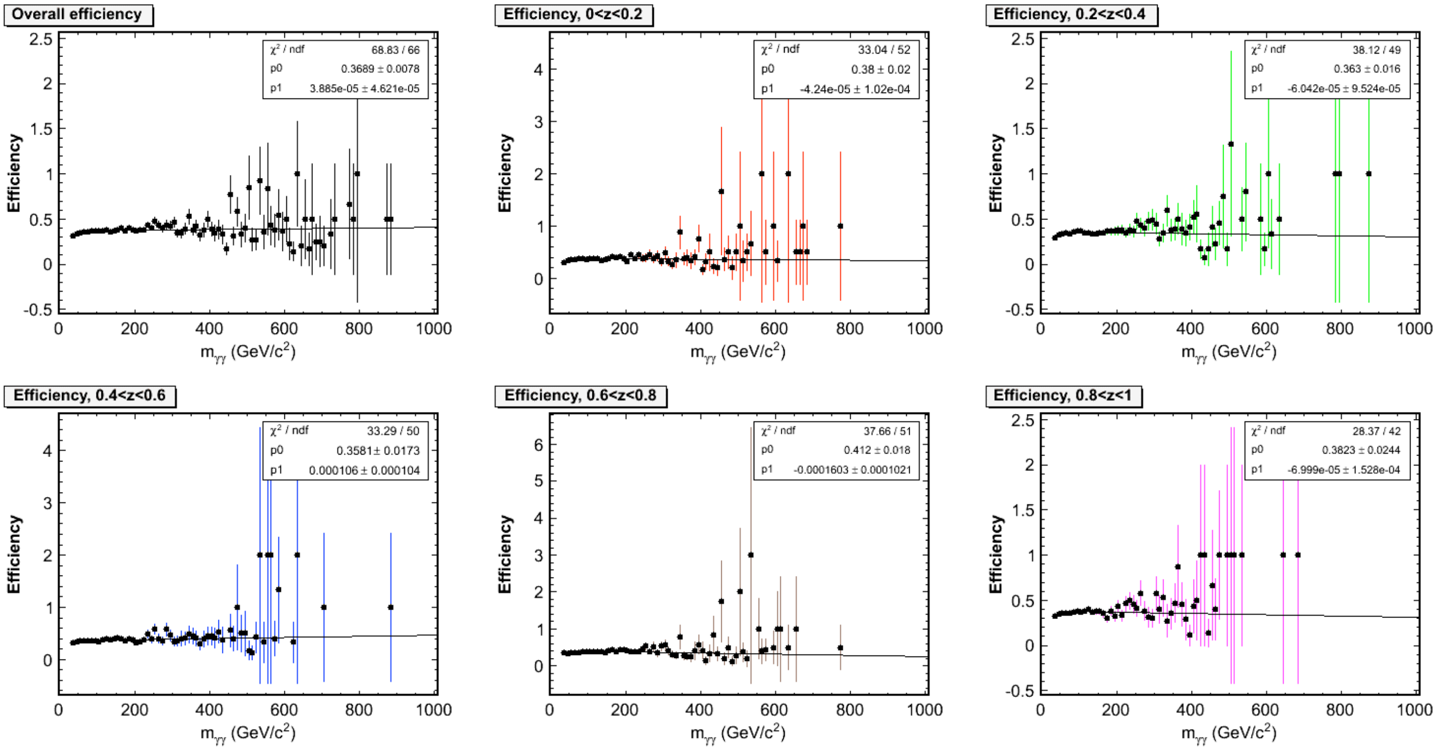

Figure 3: CDF detector efficiency as a function of diphoton mass for five regions of |*z*| and overall.

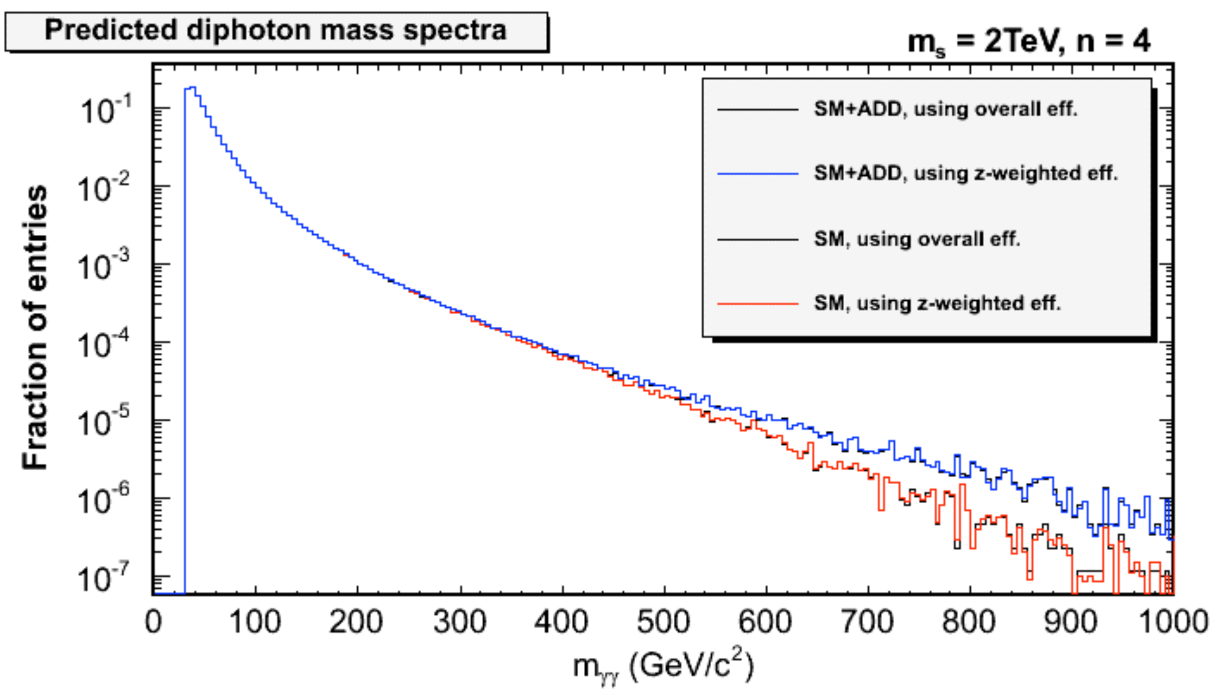

Figure 4: The diphoton mass spectra one would expect to observe at CDF, calculated using overall and z-weighted efficiency.

used to calculate the ratio of SM+ADD/SM as a function of $m_{\gamma\gamma}$, and the DIPHOX fit was multiplied by that ratio. The large quantities of simulated data gave very smooth mass spectra, so a reliable ratio could be obtained by simply dividing the two histograms. The accumulated CDF data only runs up to ~600 GeV, lower than where roughness on the ratio curve begins to occur, so any high-mass inaccuracy resulting from imperfect simulation would not affect the statistical analysis.

In order to set lower limits on the values of M_s and n, we employed the CL_s technique [9], [10]. This technique takes a set of data, a null hypothesis H_0, a test hypothesis H_1, and systematic errors for both hypotheses, and returns a CL_s value, which is defined as:

$$cls = \frac{P\ H_1\left(\Delta\chi^2_{PE} \geq \Delta\chi^2_{obs}\right)}{P\ H_0\left(\Delta\chi^2_{PE} \geq \Delta\chi^2_{obs}\right)} \tag{14}$$

with

$$\Delta\chi^2 = \chi^2(data|H_1) - \chi^2(data|H_0)$$

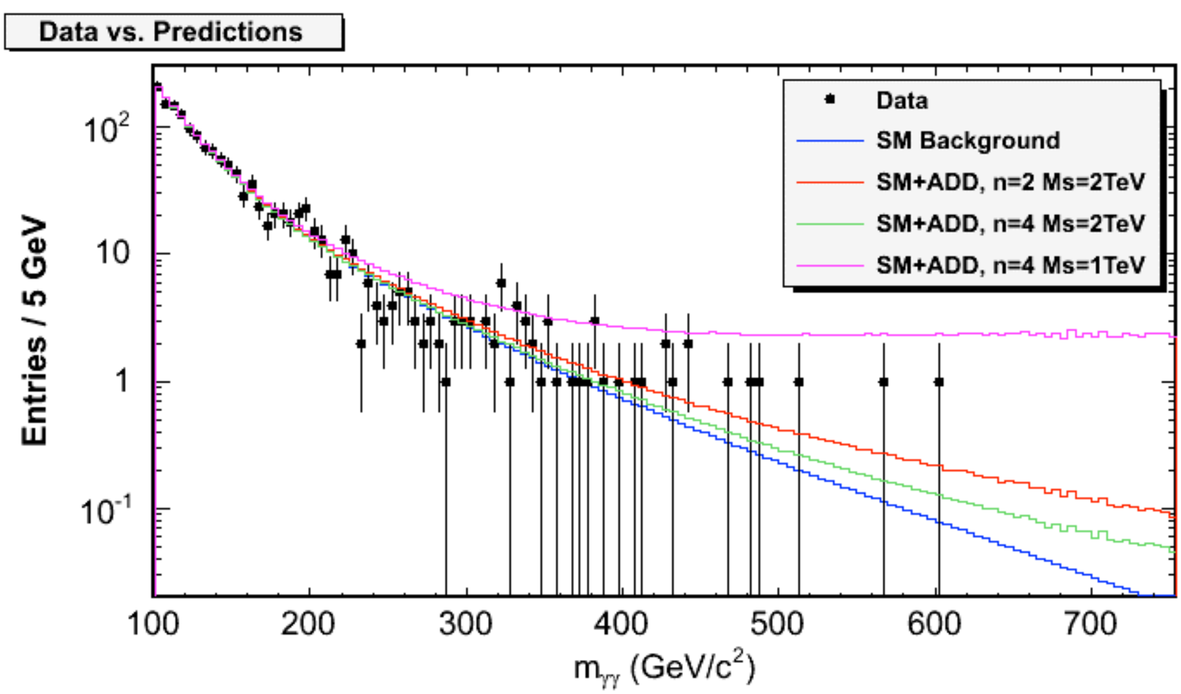

Figure 5: CDF diphoton data, plotted against the DIPHOX SM fit and various SM-ADD predictions.

A CL_s value of 0.05 corresponds to a 95% confidence level (CL). Systematic errors for the null hypothesis (SM-only mass spectrum) were previously calculated as part of the Randall-Sundrum analysis; the same calculations were applied to the test hypothesis (SM+ADD mass spectrum) on the assumption that the systematic error in that hypothesis was approximately the same. Thus, for $n = 2$ to 7, and for $M_s = 0.5$ TeV to 3.0 TeV, an expected SM+ADD mass spectrum was calculated and the CL_s value determined. For each value of n, the CL_s value was plotted as a function of M_s in 10 GeV increments, allowing us to determine the lower limit on M_s with relative ease and precision. We fit a second-order polynomial to a portion of the CL_s graph, and used the quadratic formula to ascertain where that value crossed 0.05, corresponding to a lower limit on M_s at the 95% CL.

RESULTS

Our search for the 95% confidence level lower limit on M_s for n between 2 and 7 yielded the following results, summarized in Table 1 and Figure 6:

n	2	3	4	5	6	7
M_s (obs.) (TeV)	1.76081	1.9829	1.66496	1.50268	1.39711	1.32075
M_s (exp.)	2.1564	2.11235	1.77476	1.60298	1.49189	1.40902

Table 1: Observed 95% CL lower limits on effective Planck mass, and expected lower limits.

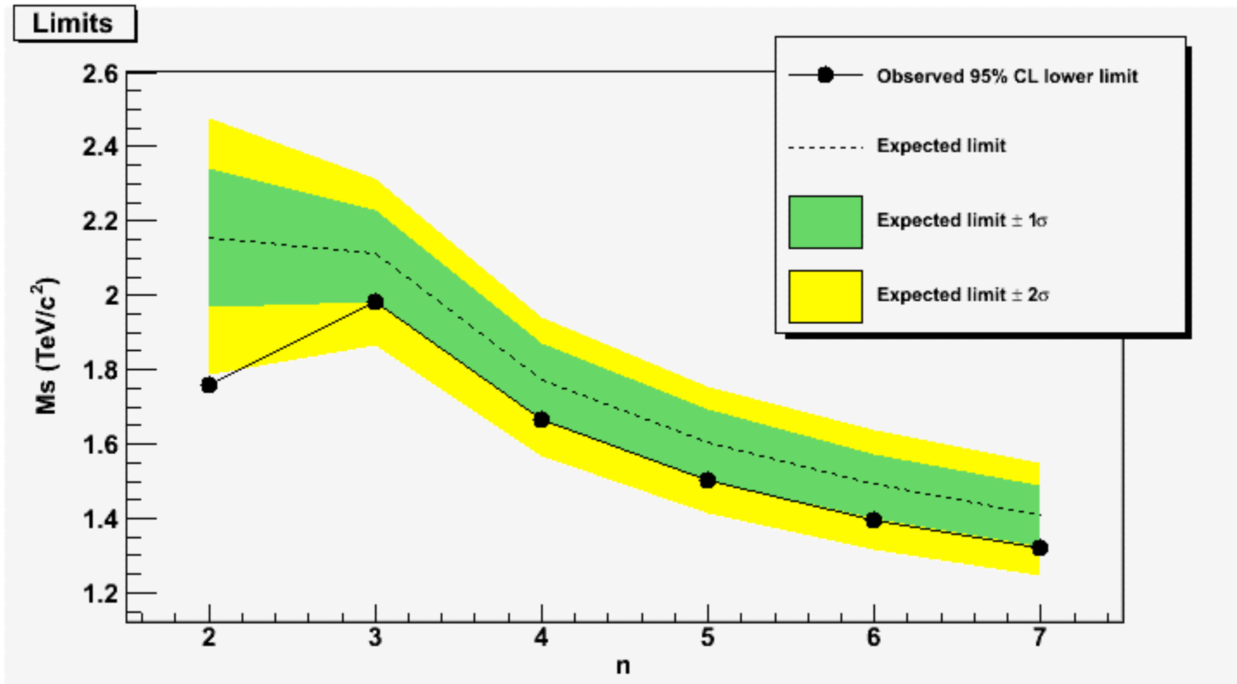

Figure 6: Observed lower limits compared against expected lower limits, with one- and two-standard deviation regions shown.

DISCUSSION/CONCLUSION

Each of our data points follow the -1σ boundary, with the exception of $n = 2$. This result is not entirely unexpected, as the $n = 2$ case generally shows the SM+ADD prediction diverging from the SM prediction at lower $m_{\gamma\gamma}$ than other cases. This $n = 2$ dip is not shown in the 2008 DØ paper [3], however, it is seen in the 2005 DØ paper that focuses only on dimuons [4]. The limits we establish are slightly, though not significantly, more stringent than those established by DØ in 2008 (Table 2). There are some notable differences in the scope of these two analyses. Our analysis takes advantage of five times as much luminosity as the 2008 DØ study (5.4 fb^{-1} compared to their 1.05 fb^{-1}). However, the 2008 DØ analysis examines both the diphoton and dielectron channels, giving them a broader range of data upon which to draw. Their analysis also uses particles detected in both the central and end-plug regions of their detector, whereas we only examine diphotons detected in the central region. DØ also used cos θ* information alongside

n	2	3	4	5	6	7
M_s (obs.) (TeV)	2.09	1.94	1.62	1.46	1.36	1.29
M_s (exp.)	2.16	2.01	1.66	1.49	1.38	1.31

Table 2: 95% CL lower limits on Ms from the 2008 D0 study [3].

diphoton mass, making their analysis two-dimensional. It is clear from this comparison that luminosity alone is not enough to significantly improve our understanding of the ADD model. While five times greater luminosity compensated for a smaller sampling channel, the combination of greater luminosity with sampling in the dielectron and dimuon channels, data from the end-plug region, and information on cos θ* should allow us to greatly improve the limits on M_s.

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