Stability of non-linear integrable systems in accelerators♪

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♪
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♪
summer student meeting♪
Fermi National Laboratory♪
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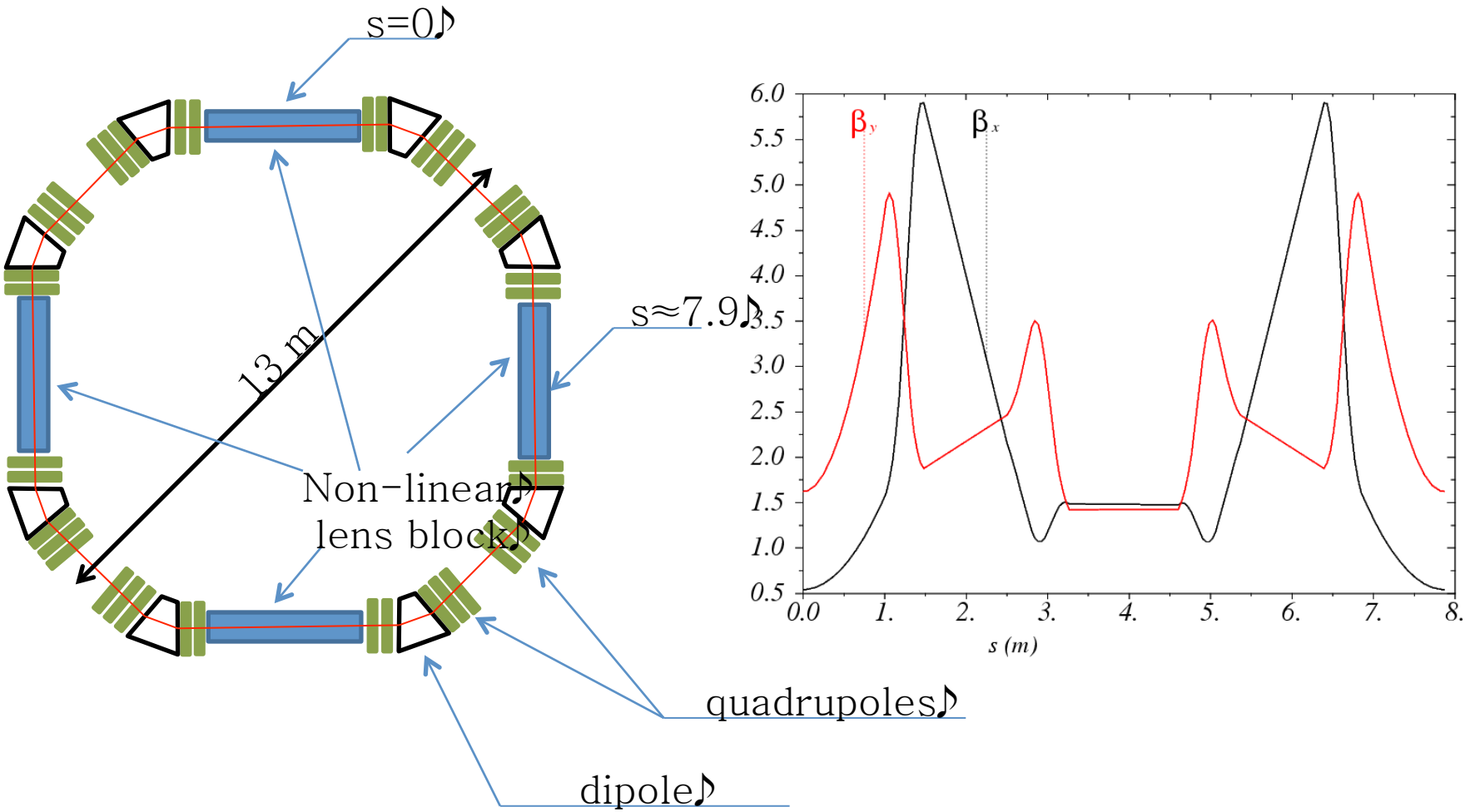
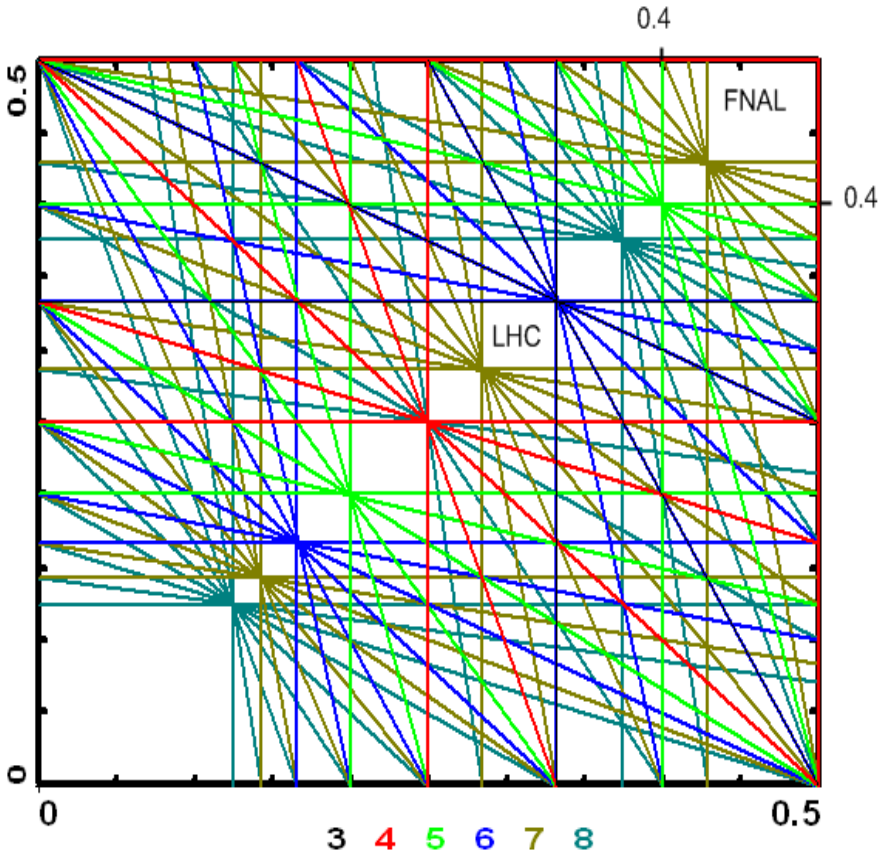
Tested non-linear system♪

by S. Nagaitsev, A. Valishev and V. Danilov♪



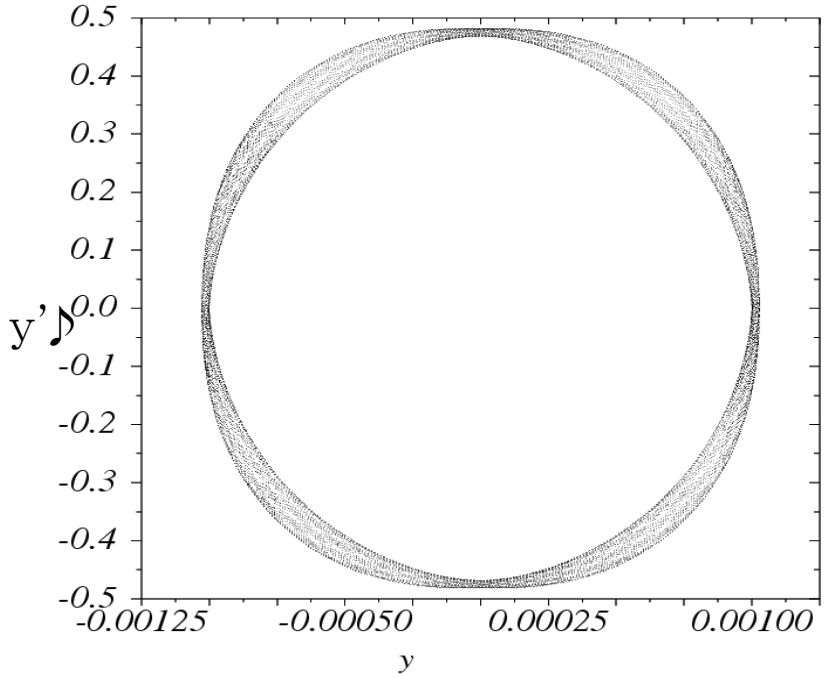
Tune spread♪



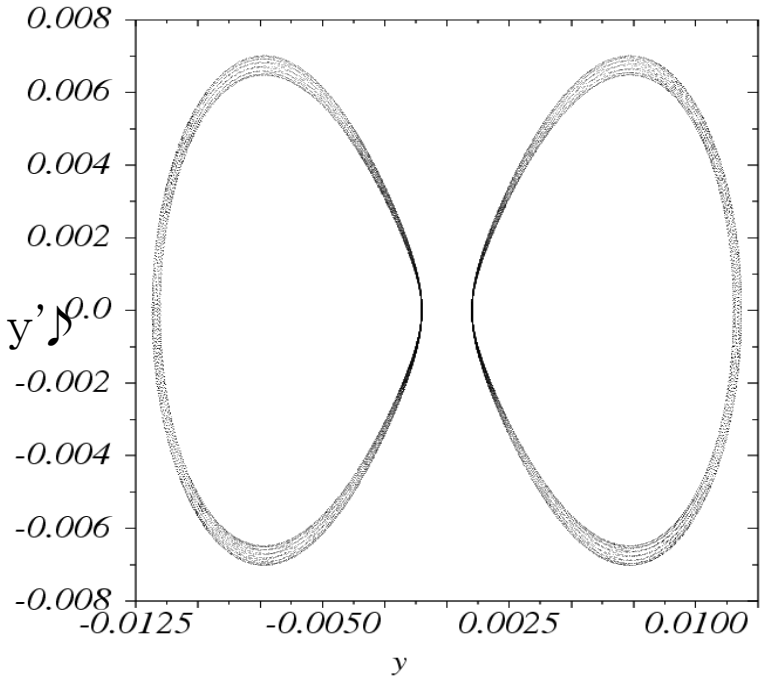
- Landau damping could be used to suppress instabilities, but it requires the wide tune spread. The larger the spread, the more stable the beam is against collective instabilities.♪
- Linear accelerators should work aside from resonances. That is why its tune spread is usually not larger than 0.1♪
- One of the main goals of creating non-linear systems is to increase tune spread♪

Particle trajectories in non-linear system♪

tn = 0.4♪



tn = 1♪



tn – strength of non-linear lens♪

If tn is bigger than 0.5, the particle trajectory does not skirt the origin of coordinates, linearized system is unstable and the beta functions cannot be found.♪

Thus the amplitudes of particles oscillations can only be found by particle tracking.♪

Main goals♪

Test this non-linear system by computer modeling with various field perturbations (sextupole, octupole e.t.c.) by following steps:♪

♪1. test linear system to measure its stable area♪

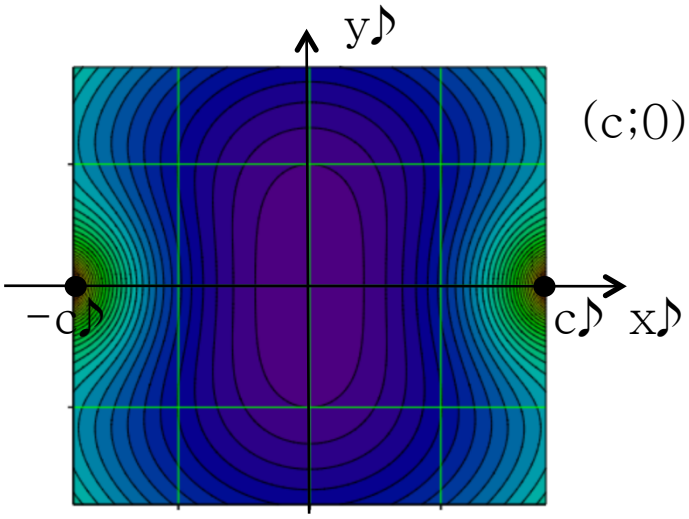
♪2. test non-linear system to measure its stable area♪

♪3. test non-linear system with sextupole and octupole perturbations ♪

♪4. make a conclusion of whether this system stable to possible field perturbations or not.♪

Aperture♪

Element♪	aperture shape♪	dimensions♪
quadrupole♪	circle♪	2 inches in diameter♪
dipole♪	rectangle♪	2x1 inches♪
non-linear lens♪	ellipse♪	x half-axis: 0.7*c*β♪ y half-axis: 7*c*β♪



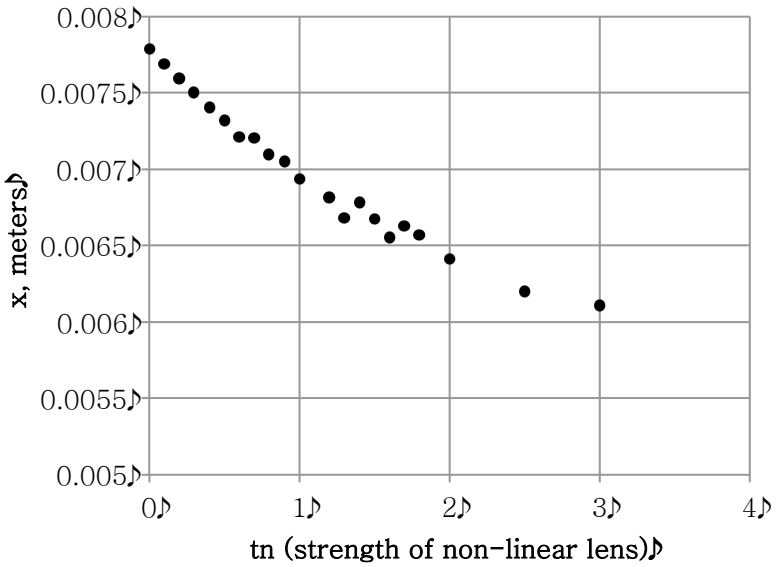
(c;0) and (-c;0) – points of singularity of the potential♩

Software: "Mad-X" with included information about non-linear lens used in the machine♪

Stable Area♪

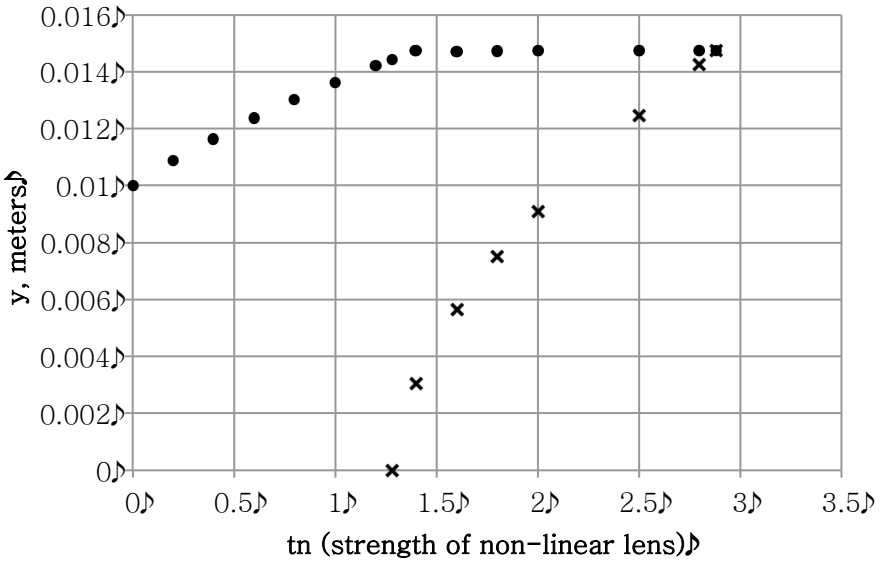
Maximal oscillations amplitude in quadrupoles = quadrupole aperture♪

y=0, py=0, px=0, x≠0♪



• max amplitude of x0 in non-linear lens♪

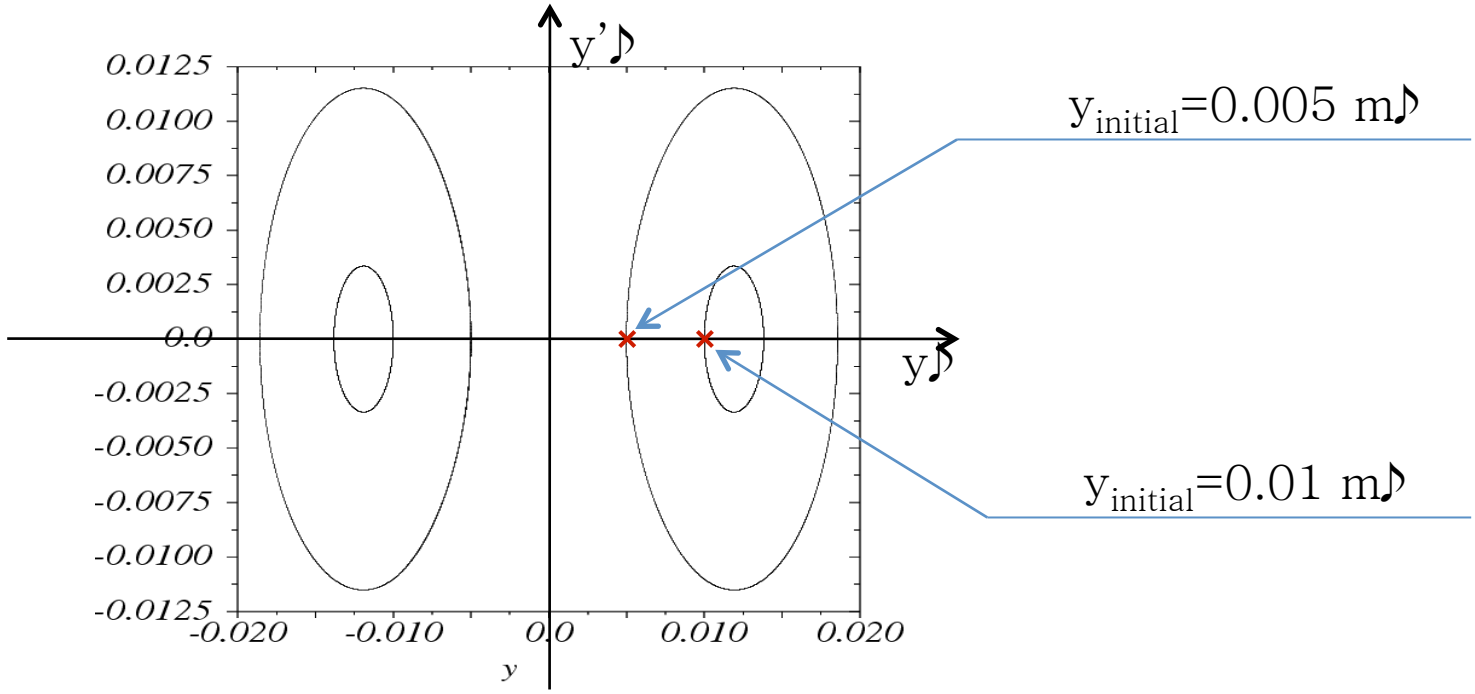
y ≠ 0, py=0, px=0, x=0♪



• max amplitude of "y" in non-linear lens♪

× min "y0" in non-linear lens♪

Repulsion from x-axis♪

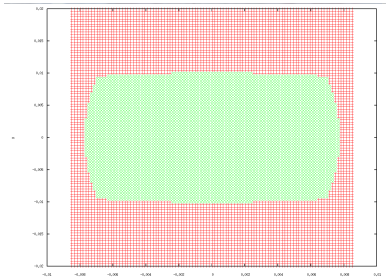


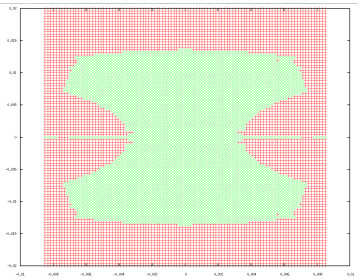
$$U = Re\left[(x+iy)^2 + \frac{2}{3} * (x+iy)^4 + \frac{8}{15} * (x+iy)^6 + \frac{16}{35} * (x+iy)^8 + \cdots\right]$$

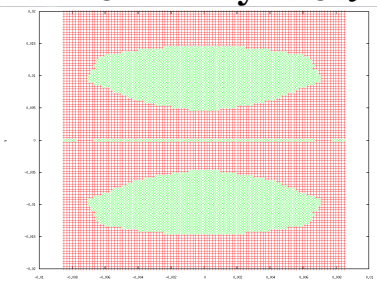
particles are pushed away from the x-axis by the field to a higher distance in (y,y') phase space♪

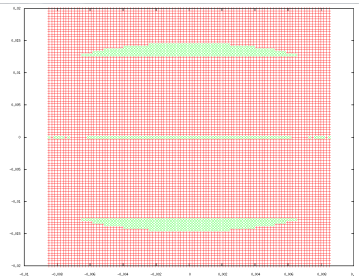
Stable Area♪

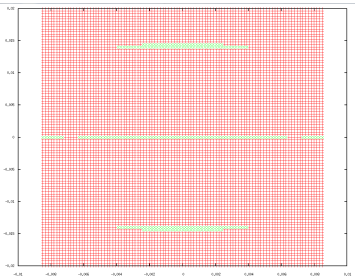
Initial coordinates of stable (green) and unstable (red) particles on y(x) plane.
x'=0 and y'=0.♪


tn=0♪


tn=0.5♪


tn=2♪


tn=2.5♪


tn=2.7♪

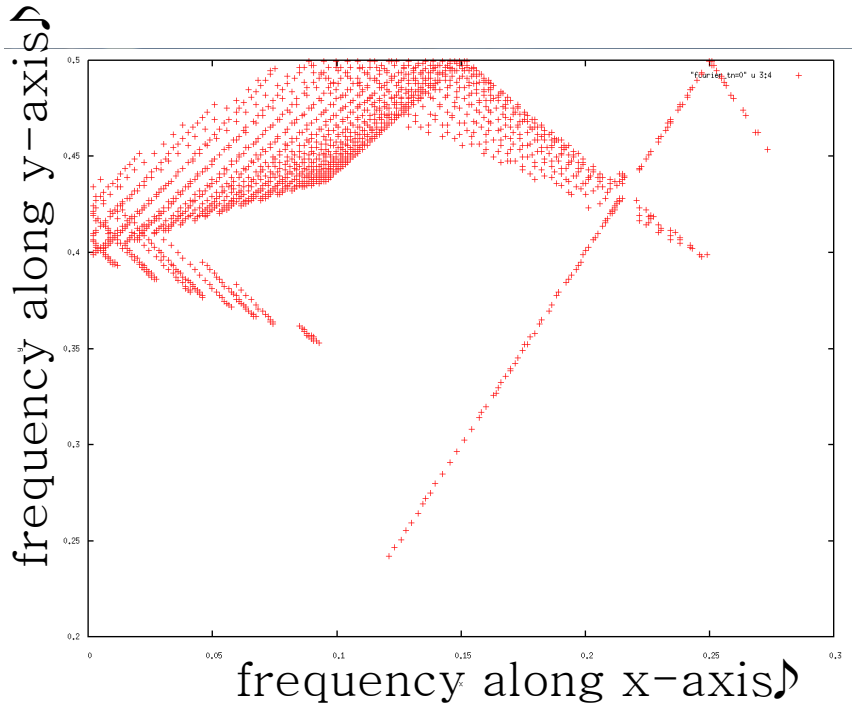
	number of lost particles			
tn	turn 1-10	turn 10-100	turn 100-1000	turn 1000-2000
0	5768	76	8	4
0,5	5545	76	11	0
1	5610	132	6	2
1,5	6943	122	8	0
2	8437	24	8	4
2,5	9625	8	0	0
2,7	10066	0	0	0

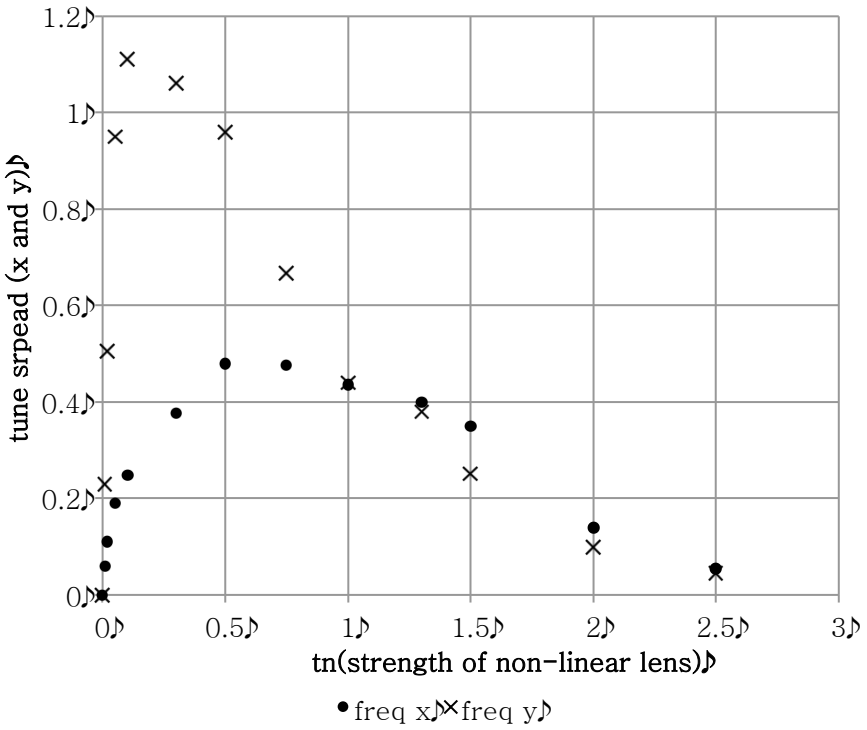
There is no dynamic aperture limit in the system♪

Fourier transform♪

For each particle:♪

$$x_i \rightarrow F(x_i)\text{♪} \Rightarrow \max(F(x_i))\text{♪}$$
$$y_i \rightarrow F(y_i)\text{♪} \Rightarrow \max(F(y_i))\text{♪}$$





Conclusions♪

1. The stable area of initial particle coordinates was analyzed as a function of non-linear lens strength.♪

♪- ♪ in a strong non-linear field, particles near the x-axis are typically lost at the physical aperture in quadrupoles♪

♪- ♪there is maximal strength of the non-linear lens for which particles survive♪

2. ♪There is no dynamic aperture limitation present in the system♪
3. Found stable area is sufficient to observe an integer resonance crossing♪

♪

Future plans (for September):♪

1. test non-linear system with sextupole and octupole perturbations♪
2. make a conclusion of whether this system stable to possible field perturbations or not♪