FEM Analysis
of
Nb-Sn Rutherford-type Cables

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09/21/2011

Introduction

The object of this study is a cable used to create superconductors. The cable is made of twenty-seven strands; each strand has a particular structure obtained combining different materials in a proper shape. The strands are twisted and then rolled by a machine to give them the cable shape. This operation causes several damages in the strands, which cause in the finished cable worst conducting properties than wished.
The target of this study is to create a FEM model to study the effects of the technology process used to manufacture superconducting cables, with the aim to understand which the critical strand is and how it will be damaged by the process.

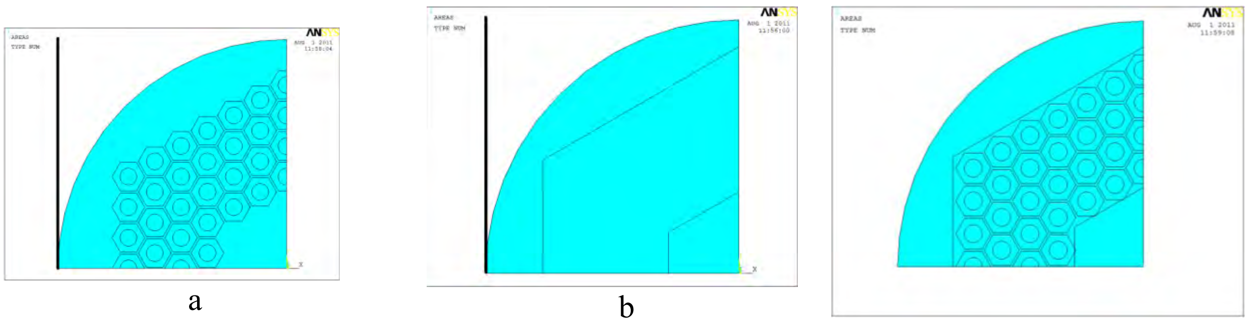
Building the model

Previews model

The superconducting R&D group of Fermilab had already realized a FEM model of a single strand. That model is really detailed: it completely represents the strand geometry and it contains all the information needed to describe the different materials position and shape.
It shows really a good match with experimental results, but because of its detailed nature, it takes really a long calculation time to find the results. This is the reason why it is not possible to use the same model to study the whole cable: the computing would take too much time to be completed.
So we decided to split the analysis in two steps: the first step will implement a simplified model of the strand to build the cable and simulate the technology process on it. With this model, we will be able to understand which strands are more interested in deformation (and so in damages) so that we can circumscribe the following analysis. In the second step, we will extrapolate information about the load cases seen by the more critical strands and then we will apply them to the detailed model of the single strand. In this way, we will significantly reduce the computing time without losing important information about the deformation process of the cable.

Simplified model of the strand

Firstly we had to decide how to simplify the strand model to let it be less heavy in calculation. Figure 1 shows the detailed model, the simplified model and a comparison between the two models. As shown in the picture, in the detailed model the strand is represented by a circle (with the mechanical properties of Cu) in which 108 hexagons are disposed in a honeycomb shape. The hexagons have Nb mechanical properties, while the little circles in them have the Sn properties.



a b c

Figure 1 – detailed model (a), simplified model (b), comparison (c)

The model represents only a quarter of the strand because of symmetry considerations. To simplify the model, we chose to represent the whole honeycomb with a single hexagon, setting for the material weighted average properties referred to area percentage of each component of the honeycomb. Table 1 shows mechanical properties used for each component of the strand.

Material	Young modulus (Gpa)	Poisson modulus	Yield tensile strength (Mpa)
Cu	110	0.364	280
Sn	41.4	0.330	20
Nb	103	0.380	420

Table 1 – Material properties

Now the model is ready to run. We wanted this simplified model to be able to give us information only about strands deformation, we didn't care about stress or material local behavior at this point. So we ran the two models (detailed and simplified) to compare deformation results. We found errors between 8-2 %. This result is not really disappointing: we are performing a non-linear analysis, so average properties were not the exact solution we needed. The simplified model stiffness appeared to be higher than detailed model stiffness. This is the reason why we chose to correct material percentages (increasing Sn percentage, which has the lowest Young modulus) with the aim to align simplified model results with detailed model results. Table 2 shows corrections applied to material percentages, while Table 3 shows the improvement obtained with the correction.

Material	Original %	Modified %	Correction %
Cu	0.206	0.156	- 0.05
Sn	0.250	0.430	0.18
Nb	0.544	0.414	- 0.13

Table 2 – material percentages corrections (1mm diameter strand)

Max y-component of displacement in simplified model (mm)		Error %
Original %	0.1183	2.2
Modified %	0.1203	0.5

Table 3 – Modified percentage results. Max y-component of displacement detected in detailed model: 0.1209 mm

Table 3 is not enough to prove the quality of our simplified model. We performed a more detailed comparison between the two model studying X and Y component of displacement, Von Mises plastic strain and plastic work. The results are shown in Figure 2

The comparison shows how good the simplified model is to represent strands deformation. Anyway, it is not appropriate to completely study the mechanic behavior of the cable: because of

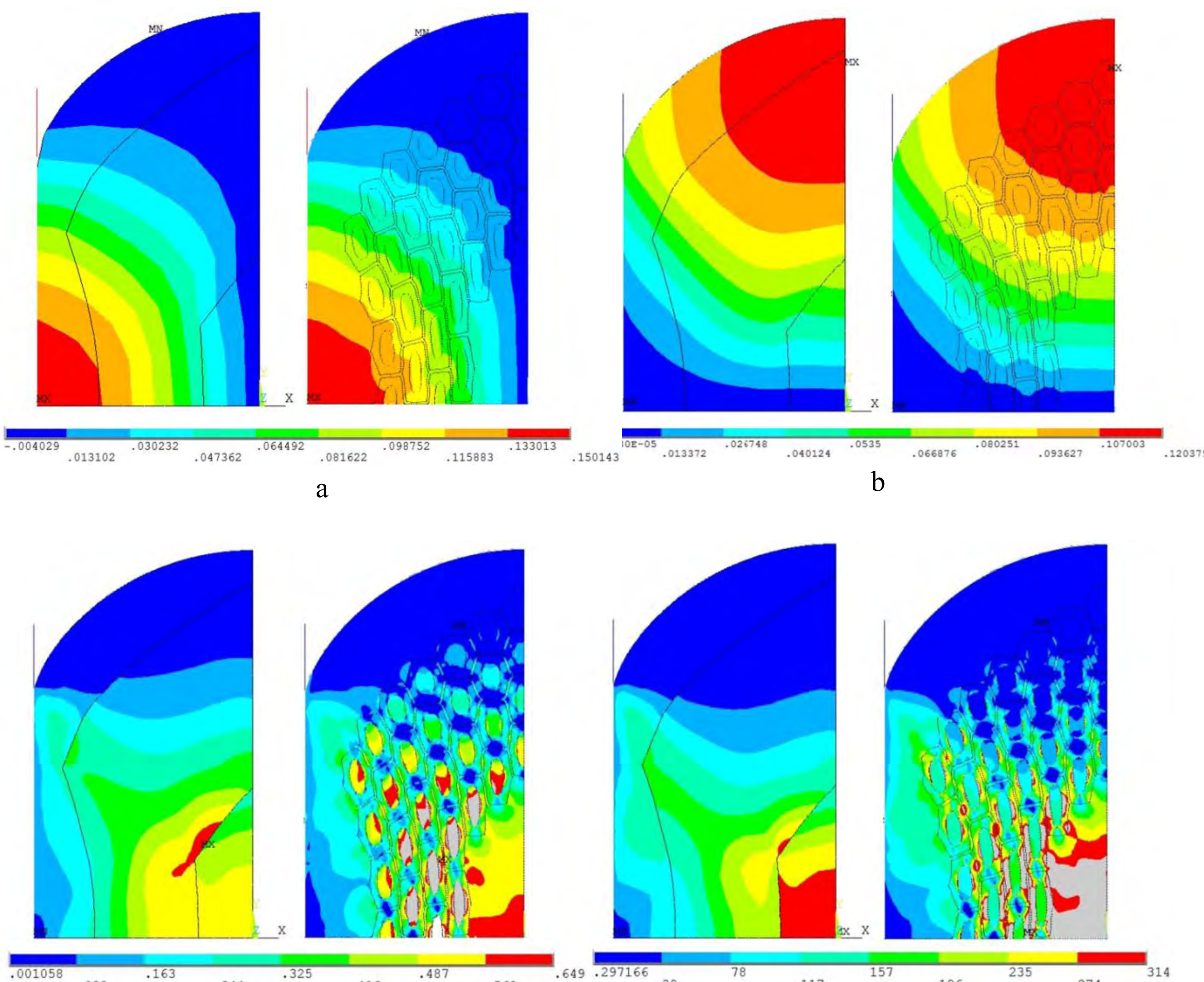


Figure 2 – Results comparison: x-displacement (a), y-displacement (b), Von Mises strain (c), plastic work (d)

its simplified geometry, this model does not estimate local effects as the detailed model does. So the idea is to use the simplified model to build an entire cable model which need short calculus time to run. With this model, we can find information about strands deformation and load cases. Then, using those results as an input, we can model a single strand in a detailed way applying the loads found in the simplified analysis.

Cable model

The main target of this work is to study a twenty-seven strands cable. Anyway, we wanted to find out something about the effects that the number of strands (N) has on cable behavior. So we build several cable models for each N analyzed. Figure 3 represents the twenty-seven strands cable model (symmetry considerations allowed us to simplify the model).

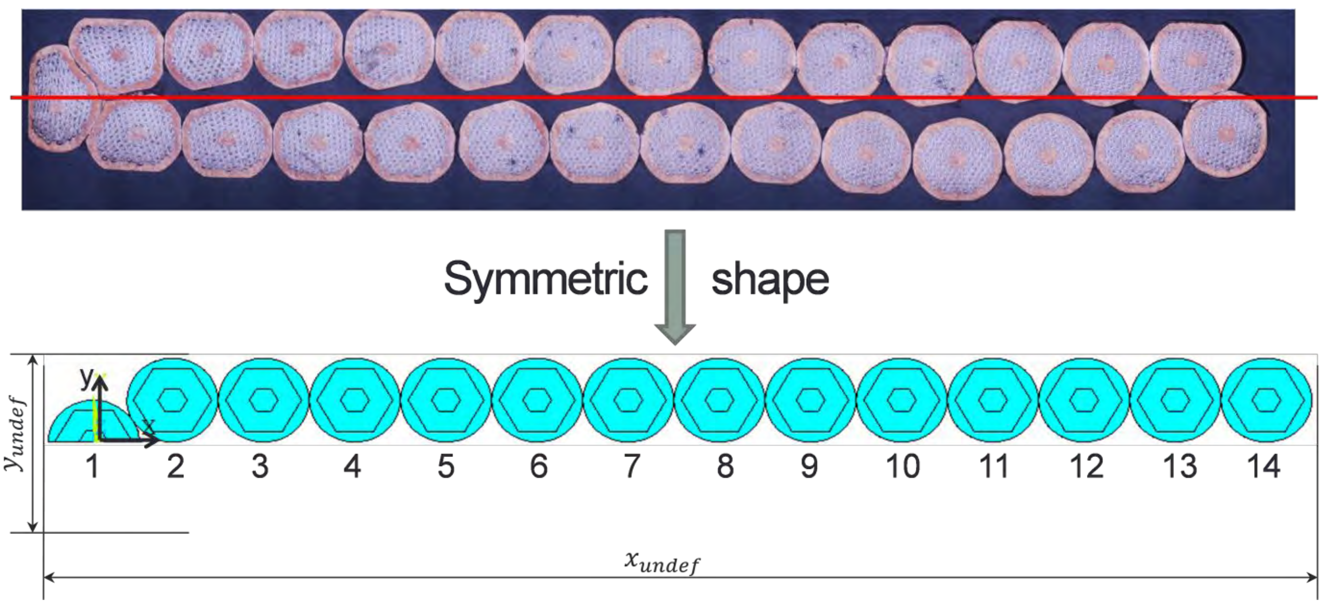


Figure 3 – Twenty-seven strand cable model

It is possible to find several different configurations of the strands along the cross section of the cable. We decided to study the configuration shown in Figure 3 because left edge (where strand one is located) is different from right edge (where strand fourteen is located); in this way we can study both the situation at once.

Load cases

The cable is deformed by 4 rollers which give it the final shape, so we decided to set an imposed displacement value (calculated knowing the distance between corresponding rollers in the process). The deformation is given in 2 steps:

1) Rectangular deformation: all the strands have the same y-deformation (Figure 4)

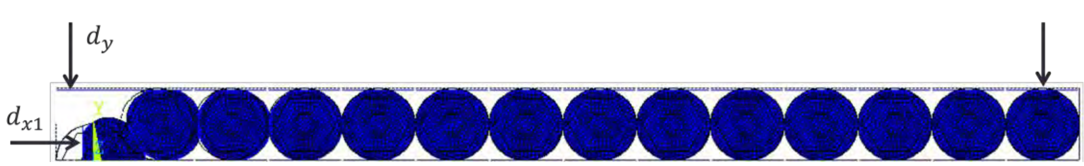


Figure 4 – rectangular deformation

2) Keystoned turk-head: strands have different y-deformation (Figure 5)

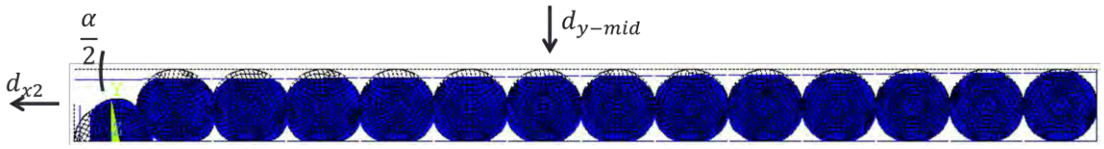


Figure 5 – Keystoned turk-head

In order to find out the displacement values to be implemented in the model, we had to know the undeformed measures of the cable. Geometric considerations allowed us to write two formulas for x and y underformed measure (Figure 6).

$$r' = \frac{r}{\cos(\alpha_{pitch})}$$

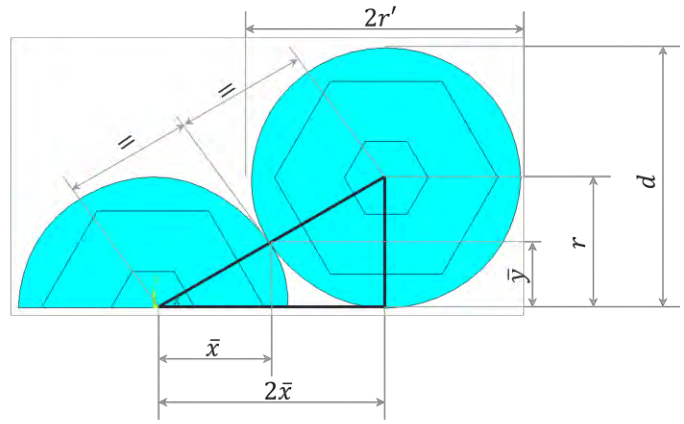


Figure 6 – Geometric considerations

Knowing the ellipse equation (1) and considering the black triangle (2), we can write:

1) $$\frac{x^2}{r'^2} + \frac{y^2}{r^2} = 1$$

2) $$\frac{r}{2\bar{x}} = \frac{\bar{y}}{\bar{x}}$$

Solving the system we find equations (3), (4) and (5):

3) $$\bar{x} = r'\frac{\sqrt{3}}{2}$$

4) $$x_{undef} = 2\bar{x} + (N-1)r' = \left(\sqrt{3} + N - 1\right)r'$$

5) $$y_{undef} = 2d$$

Because of symmetry considerations, we know that the displacement in y direction must be equally divided between upper and lower edge of the cable (equation (6)).

6) $$d_y = \frac{y_{undef} - rollers_{y-distance}}{2}$$

Considering displacement in x direction, we can't know how to exactly apply the load, so we decided to consider two different cases:

A) Whole displacement applied on left edge of cable ($split_{factor} = 1$ in equation (7))
B) Displacement equally distributed between left and right edges of cable ($split_{factor} = 2$ in equation (7))

$$\textbf{7)} \qquad d_x = \frac{x_{undef} - rollers_{x-distance}}{split_{factor}}$$

Figure 7 shows the differences between load cases (A) and (B).

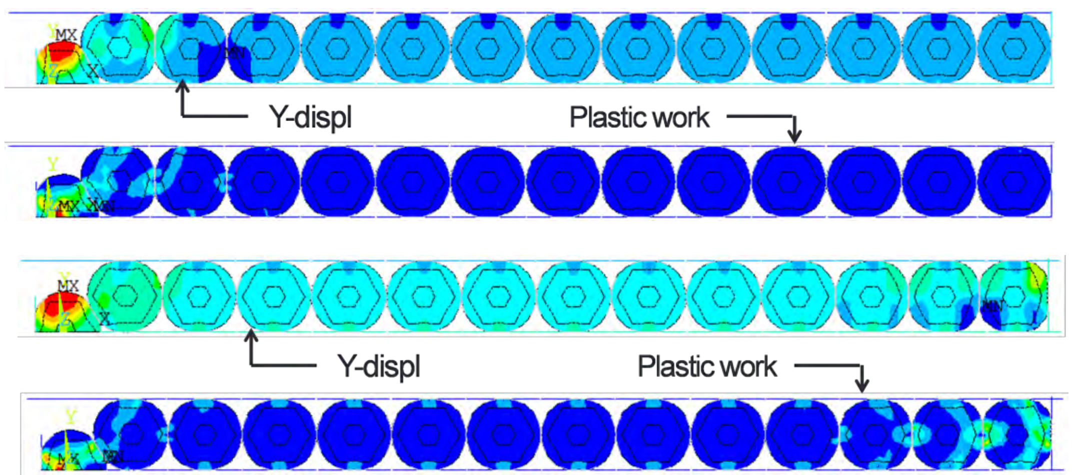


Figure 7 – Load cases: $split_{factor} = 1$ (top) and $split_{factor} = 2$ (bottom)

Using the model

Deformation as a function of N

For each N analyzed, we chose to apply 3 values of $rollers_{x-distance}$ (w_c):

1) $w_c = x_{undef} - \Delta x$ (with same Δx for all N)
2) $\frac{x_{undef} - w_c}{x_{undef}} = a$ (with same a for all N)
3) $w_c = \frac{N}{2} \frac{d_{strand}}{\cos(\alpha_{pitch})}$ (which is the commonly used formula in cables design)

We applied both load cases ((A) and (B)) in order to find out how the number of strands changes the deformation condition in the cable. Because of the cable shape and because of the load cases, we have that strand one has the highest displacement in y direction. We thought that maximum value of y-displacement in strand one would be the correct parameter to study deformation level in the cable: the higher is maximum y-displacement in strand one, the higher is cable deformation and failure probability. Figure 8 confirms that strand one has the highest y-displacement (red areas feel 99% of the maximum value of y-displacement):

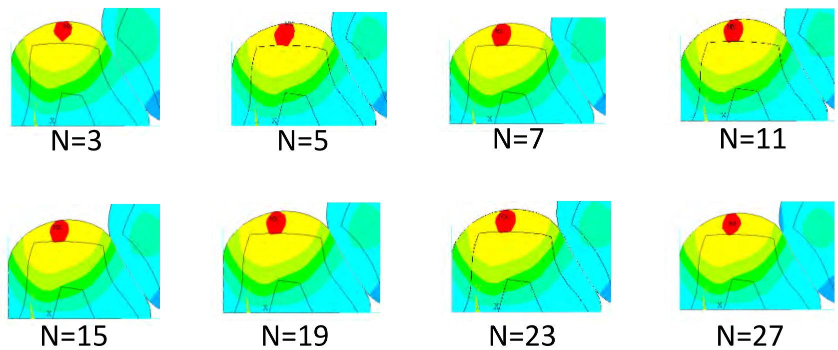


Figure 8 - Max y-displ location

Figure 9, Figure 11, and Figure 10 show how maximum y-displacement value changes with N:

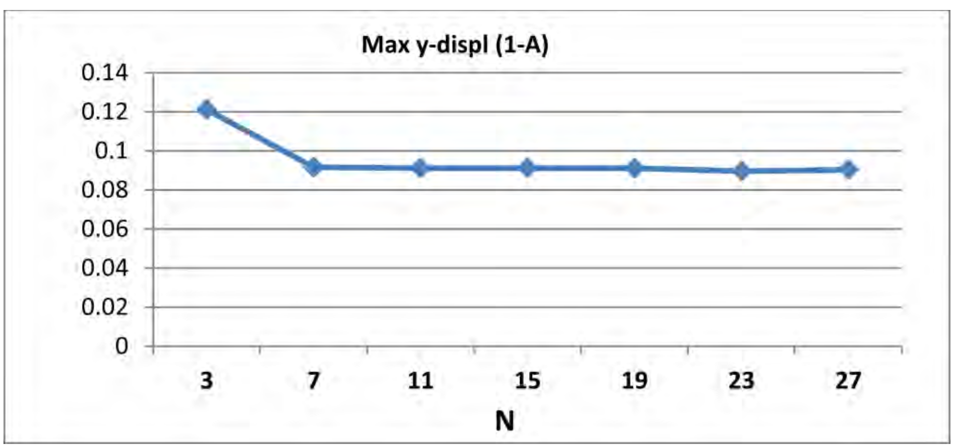


Figure 9 – Load case 1-A

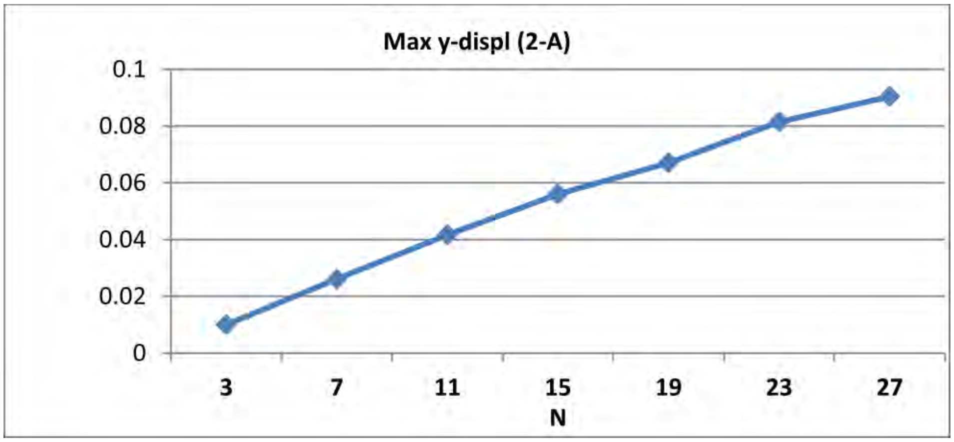


Figure 11 – Load case 2-A

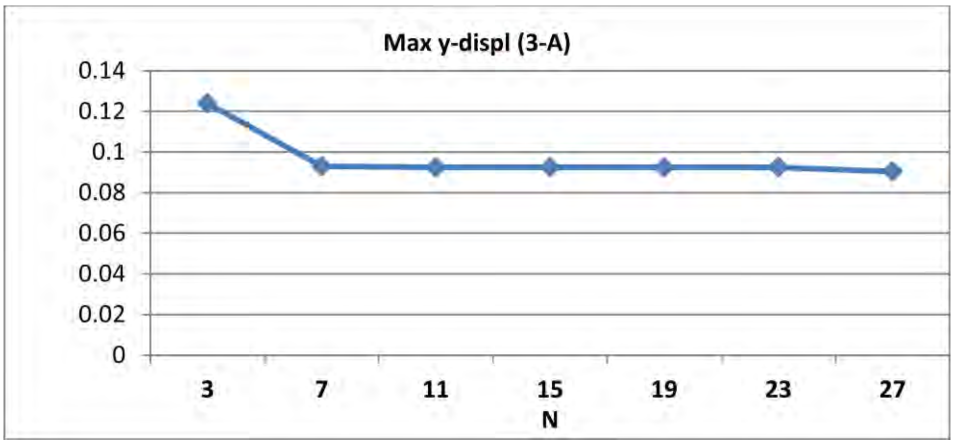


Figure 10 – Load case 3-A

Firstly we noticed that load case (1) and (3) are really similar, so we decided to keep only load case (1) for next analysis. Then we saw that the maximum y-displacement has the same trand of the load aplyed: if d_x increases linearly with N, maximum y-displacement also increases linearly with N; otherwise, if d_x is the same for all N, maximum y-displacement is constant.
Load case (B) shows that load repartition is important in maximum y-displacement value, but it doesn't change its relation with N (Figure 12 and Figure 13).

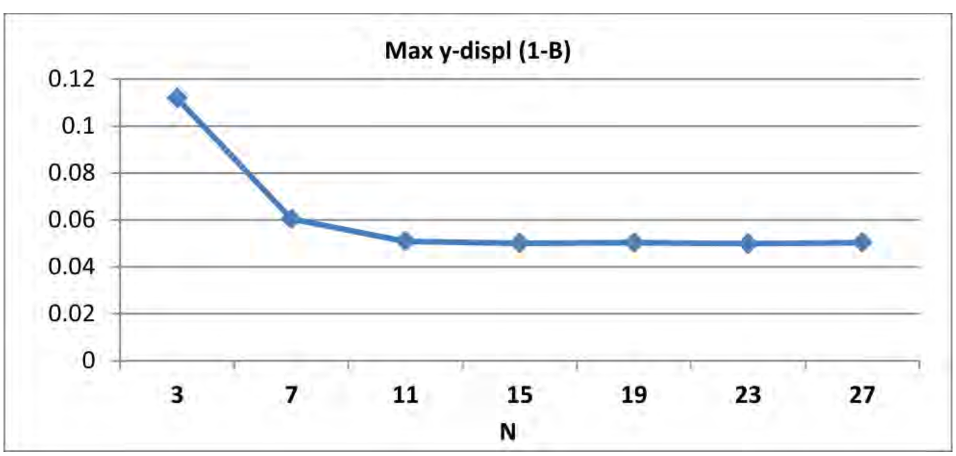


Figure 12 – Load case 1-B

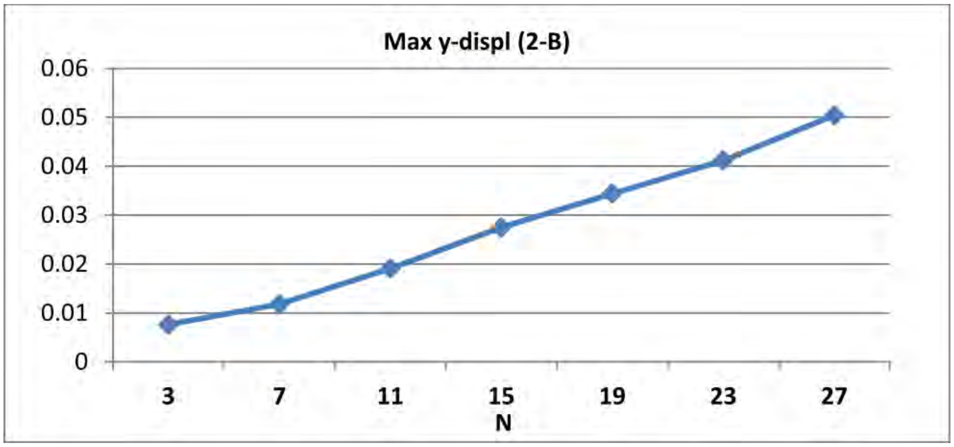


Figure 13 – Load case 2-B

Choosing odd or even N

We found that N has not a big influence on cable solicitation as long as you give it a constant value of w_c and you consider a cable with an odd N. We wanted to be sure that this consideration is true also for an even N. Figure 14 shows cable's configurations which have the highest values of x_{undef} with N=9 and N=10. As shown, choosing an even N gives to the cable two symmetry planes; this is the reason why we are allowed to reduce the model to a quarter of the total cable, applying just a half load to left edge. This suggested us that an even N could cause a smaller load on the cable, so we studied both cases (odd or even N) to understand which one is better.

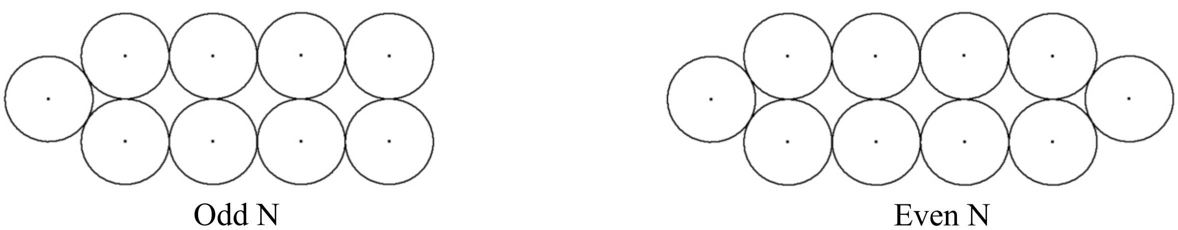


Figure 14 – Undeformed cable with an odd (left) and an even (right) N

Using the formula: $w_c = \frac{N}{2}\frac{d_{strand}}{\cos(\alpha_{pitch})}$, we built Table 4 (numbers contained in the table are normalized by r'). Last columns of the tables contain the value of displacement to be applied to the model (d_x). As shown in the table, with an odd N we have a load which is exactly a half of the load given by an even N. On the other hand, with an even N we can apply half load to one edge of the cable. Therefore the choose of an even or an odd N changes x_{undef} and at the same time it also changes w_c, so that overall it doesn't really change load condition on the cable.

Odd N	x_{par}	w_{c-par}	Δ_{w-par}
3	3.732051	3	0.732050808
5	5.732051	5	0.732050808
7	7.732051	7	0.732050808
9	9.732051	9	0.732050808
11	11.73205	11	0.732050808
13	13.73205	13	0.732050808
15	15.73205	15	0.732050808
17	17.73205	17	0.732050808
19	19.73205	19	0.732050808
21	21.73205	21	0.732050808
23	23.73205	23	0.732050808
25	25.73205	25	0.732050808
27	27.73205	27	0.732050808
29	29.73205	29	0.732050808
31	31.73205	31	0.732050808
33	33.73205	33	0.732050808
35	35.73205	35	0.732050808
37	37.73205	37	0.732050808
39	39.73205	39	0.732050808
41	41.73205	41	0.732050808

Even N	x_{par}	w_{c-par}	Δ_{w-par}
2	3.464102	2	1.464101615
4	5.464102	4	1.464101615
6	7.464102	6	1.464101615
8	9.464102	8	1.464101615
10	11.4641	10	1.464101615
12	13.4641	12	1.464101615
14	15.4641	14	1.464101615
16	17.4641	16	1.464101615
18	19.4641	18	1.464101615
20	21.4641	20	1.464101615
22	23.4641	22	1.464101615
24	25.4641	24	1.464101615
26	27.4641	26	1.464101615
28	29.4641	28	1.464101615
30	31.4641	30	1.464101615
32	33.4641	32	1.464101615
34	35.4641	34	1.464101615
36	37.4641	36	1.464101615
38	39.4641	38	1.464101615
40	41.4641	40	1.464101615

Table 4 – Deformation to be applied to a cable with an odd (left) or an even (right) N.

Comparison with experimental results

In the paragraph “Load cases” we saw that it is not possible to know how the load is distributed between left and right edges of the cable (with an odd N). In order to understand which load case best suits the real load condition, we performed a comparison between experimental and FEM analyses results. Figure 15 shows the measure used in the comparison for the twenty-seven strands cable.

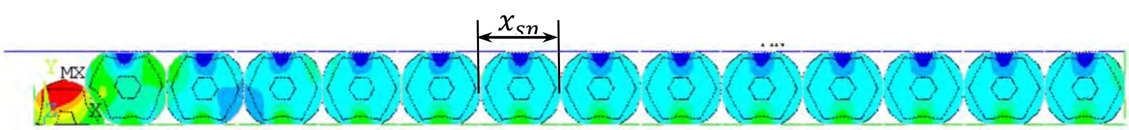


Figure 15 – Measure used for the comparison

Comparison results are shown in Figure 17 and Figure 16 for both load cases A (whole load on left edge) and B (load equally divided between left and right edge).
As shown in the second graph, in load case B the FEM model’s lasts strands result more deformed than on the experimental measurements.

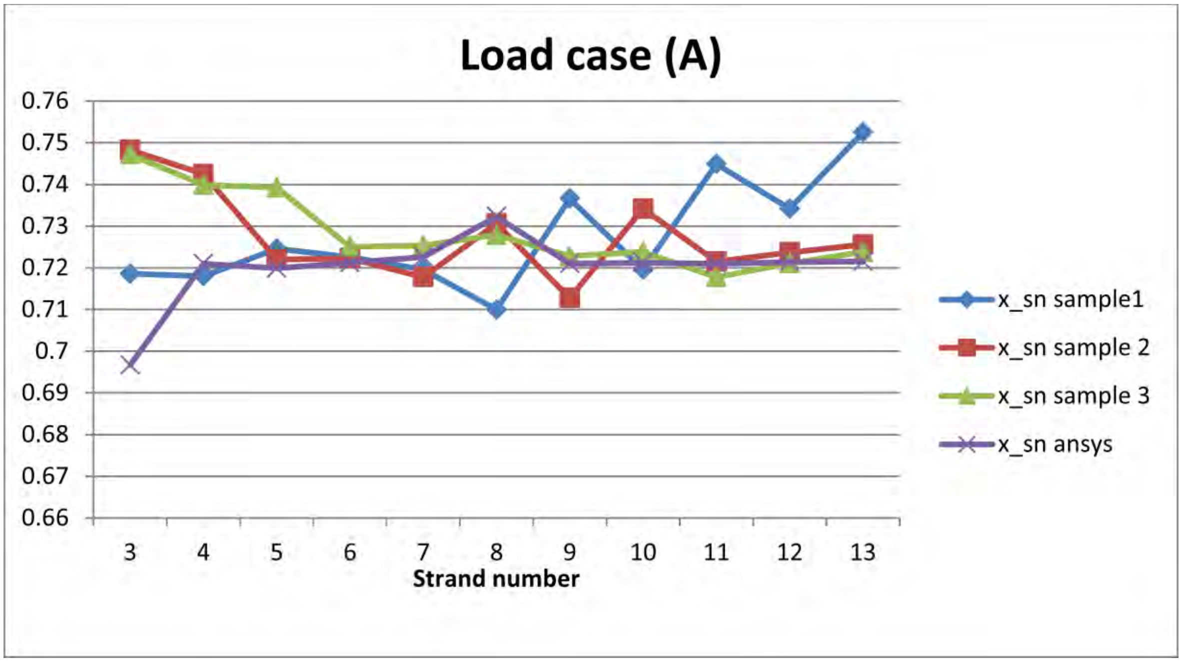


Figure 17 – Measure comparison for load case A

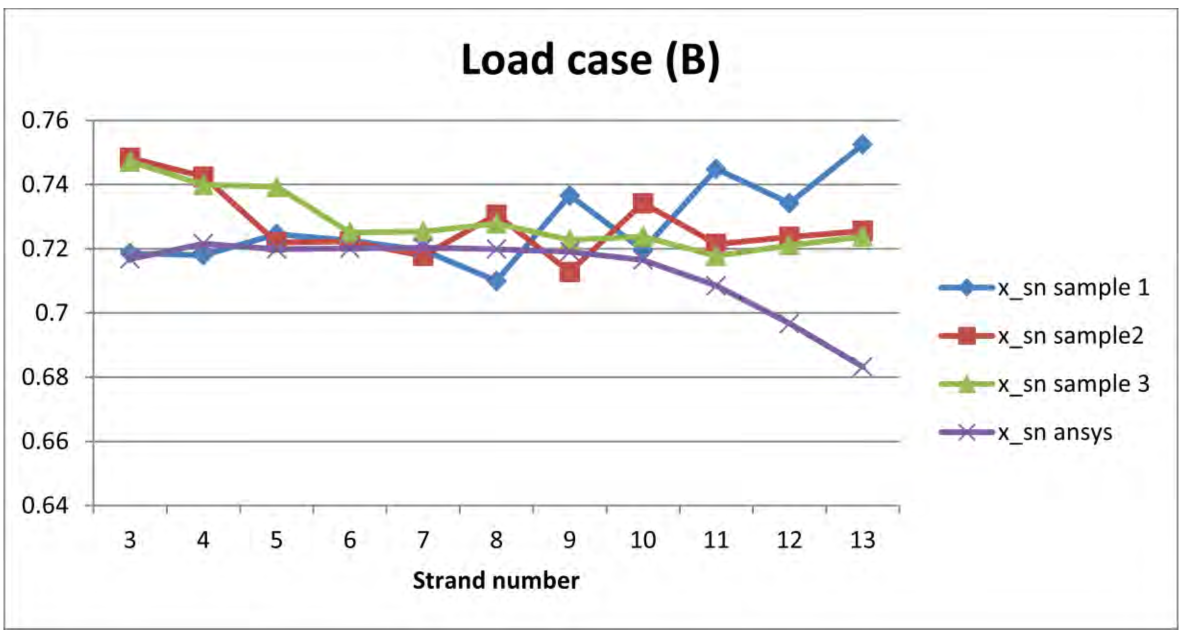


Figure 16 – Measure comparison for load case B

Figure 18 shows the overlapped picture of strand one and fourteen (obtained with the model analysis and with a microscope) in both load cases A and B. This picture confirms that load case A best represents load distribution in the cable.

Figure 18 – Overlapped picture of strand 1 and 14 in load cases A (left) and B (right)

It is possible to understand this result thinking about the strand as a spring. Figure 19 shows a schematization of the cable: three springs are represented instead of the strands which should be in left and right edges.

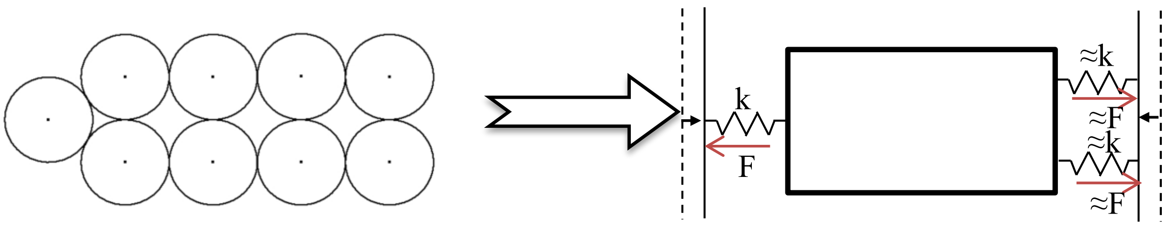


Figure 19 – Schematic view of the cable

Trying to impose the same displacement on both edges (as load case B should do), springs will react with a different value of force (there are two springs on right side, each one reacting with about the same force then the single spring on left side). So the cable (which is free to move in the plane as soon as the spindle is not rigidly fixed on the machine: it could orientate itself for small angular values) will move in left direction, and the less rigid part of the structure will get the highest deformation.

Detailed model of the critical strands

Thanks to the whole cable analysis, we could determine which the critical strands were. The first step of rectangular deformation gives the highest values of plastic energy to the first strand of the cable. The keystoning deformation increases the areas with high values of plastic energy but doesn't really change the solicitation in strand number one. On the other hand, the keystoning mainly acts on the strand number two, so that it could become the critical strand. Figure 20 shows critical strands in rectangular and keystoning deformation.

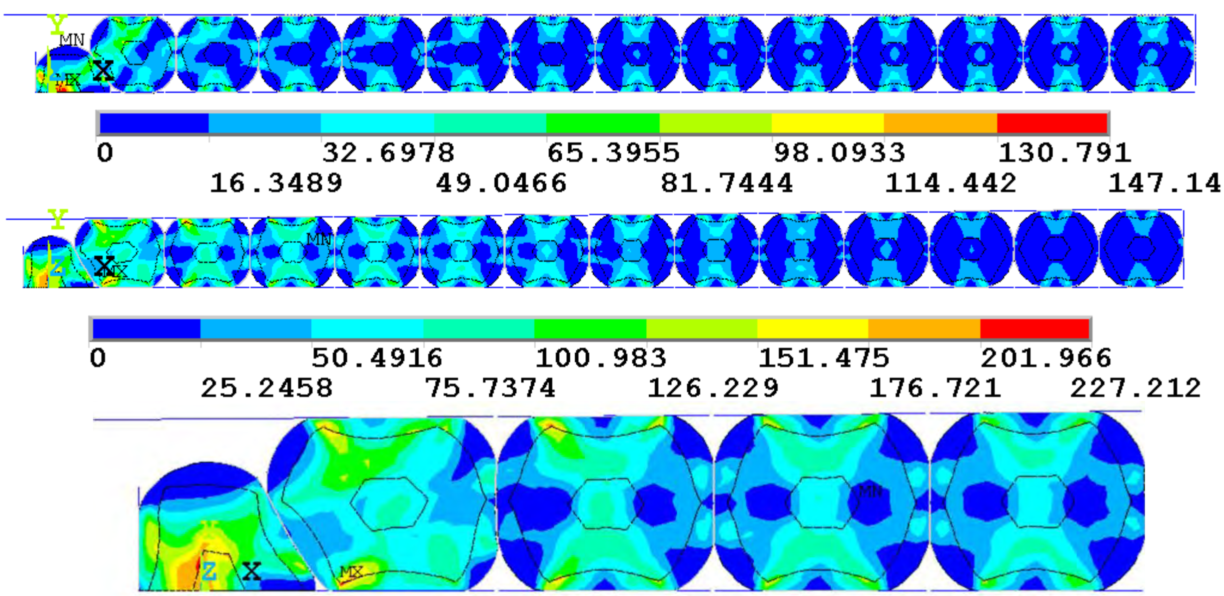


Figure 20 – Plastic work in rectangular (top) and keystoning deformation (middle). Zoom of critical strands (bottom)

Results shown in Figure 20 are obtained modeling a twenty-seven strands cable with load data stated in Table 5.

d_{x1} (mm)	d_y (mm)	d_{x2} (mm)	d_{y-mid} (mm)	α (deg)
0.242	0.05	0.054	0.085	1

Table 5 – Load data for the twenty-seven strands cable

These analyses let us understand that the most critical strand for rectangular deformation is strand number one; this is confirmed by the experiments which have all of the damaged subelements in strand one. So we performed the analysis on a detailed model of strand one, by using surface displacements obtained through the simplified cable model.

We had two different cables made with different N and different load cases:

- Twenty-seven strands cable ($d_{x1} = 0.242\ mm, d_y = 0.05\ mm$)
- Forty strands cable ($d_{x1} = 0.428\ mm, d_y = 0.05\ mm$)

Results are shown in and Figure 21 and Figure 22.

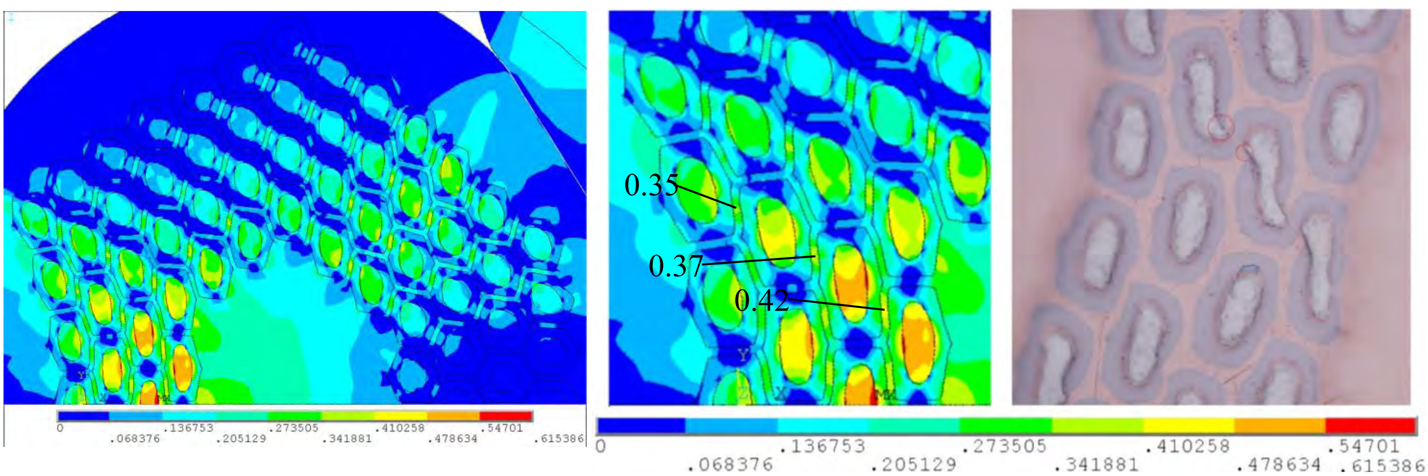


Figure 21 – Principal traction strains in strand 1 of the 27 strands cable

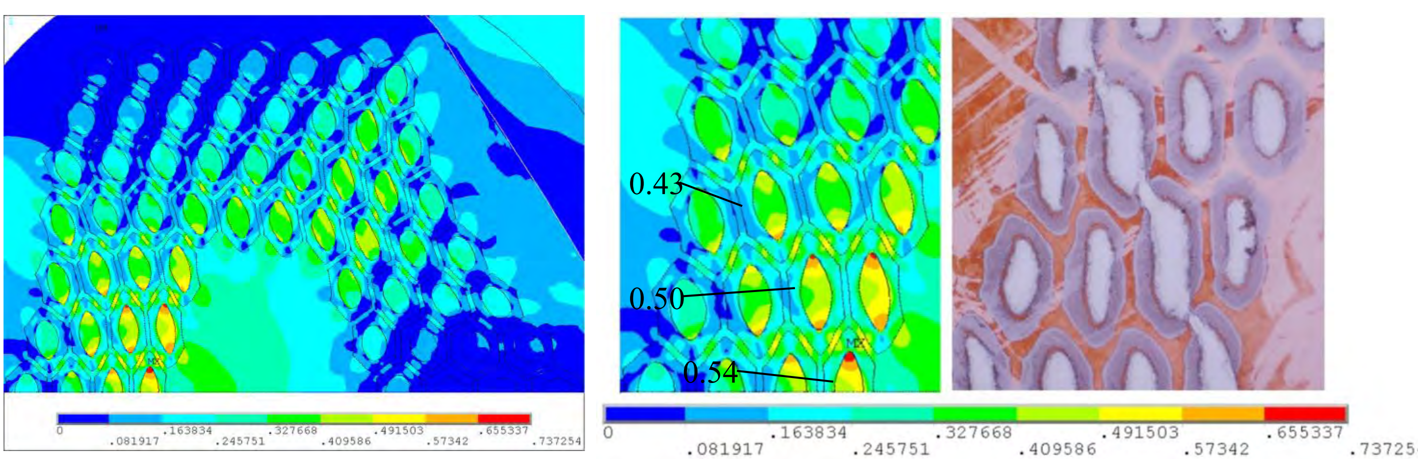


Figure 22 – Principal traction strains in strand 1 of the 40 strands cable

In the most critical area (where some experimental samples show damaged subelements) the principal traction strain reaches values between 0.35 (Figure 21) and 0.53 (Figure 22), which is the same interval detected by a previews work on the single strand deformation (0.48±0.1).
Obviously we would need a larger specimen to really compare experimental results, but we can use these preliminary results to study model's sensitivity to some parameters (as rollers' x and y distance).

Sensitivity to rollers' x distance

In order to understand how rollers' x distance acts on the cable, we performed several analyses on the detailed model of the first strand with different values of w_c. The commonly used parameter for this kind of study is called width compaction (w_{comp}, defined in equation (11)). Anyway, w_c is not really a good parameter for this study because it has different effects if applied to cables with different diameters and different N.
Looking for a more general parameter, we noticed that using $w_c = Nr'$, it is possible to obtain a Δ_w which is a constant for all N (see Table 4). To preserve this property also for different values of rollers' x distance, we decided to express w_c as stated in equation (8).

8) $$w_c = (N + \gamma)r'$$

In this way it is still possible to have a constant Δ_w for all N, as show in equation (9) (using equation (4) and (8)):

9) $$\Delta_w = x_{undef} - w_c = \left(\sqrt{3} - 1 - \gamma\right)r'$$

At this point it is possible to eliminate the dependence from r' by using an adimensional parameter (equation (10)):

10) $$\varepsilon_w = \frac{\Delta_w}{r'} = \sqrt{3} - 1 - \gamma$$

11) $$w_{comp} = \frac{w_c}{x_{undef}}$$

We investigated a range of γ between -0.3 and 0.5. To understand strand one's behavior, we looked for high values of principal traction strain in the copper channels between subelements, in order to find out if critical areas in the strand can change depending on load cases. Figure 23 shows how load are obtained from the simplified model of the cable to be applied to the detailed model.

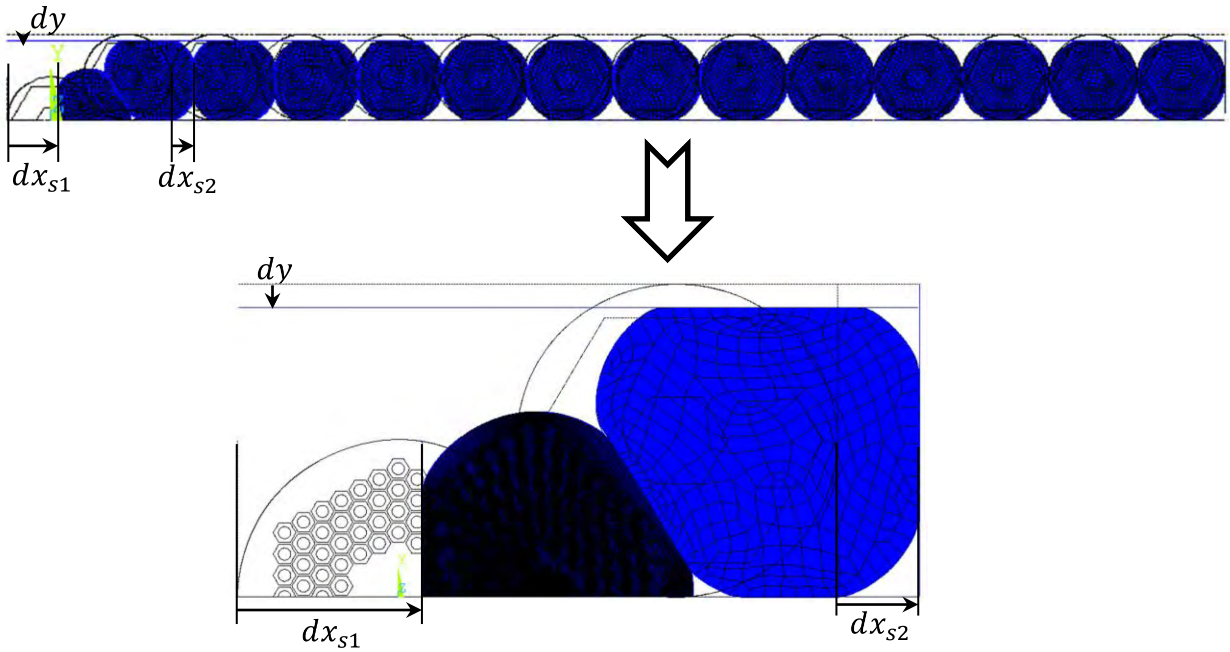


Figure 23 – Load application on detailed model

Table 6 shows the load values obtained using different γ.

γ	dx_{s1}	dx_{s2}	dy	Width compaction
-0.3	0.416	0.183	0.052	0.96
0	0.262	0.094	0.052	0.98
0.5	0.1265	0.030	0.052	0.99

Table 6 – Load cases for various γ

We mapped the principal traction strain in the copper spaces between subelements, in order to find out which the critical areas were and to study γ's influence on their location. The map used for this study is shown in Figure 24. Strains are collected from row one to row thirty; yellow rounded rows were not used because of really low strain values. In each row, values are collected from inner to outer points.

All the strain values located in the strand have been ordered with this criterion in a vector. It is possible to view strains trend in strand one for each γ plotting that vector as shown in Figure 25. We can analyze left and right part of the strand separately. In left part we find two areas with high values of strain (circled in orange and black in Figure 24 and Figure 25) while in left part we can notice just one critical area (circled in green in Figure 24 and Figure 25). As load increases, critical areas expand and max values become higher. The trends in Figure 25 show that increasing the load value, right part of strand one gradually becomes more critical then left part, so that damage can be expected also in those subelements.

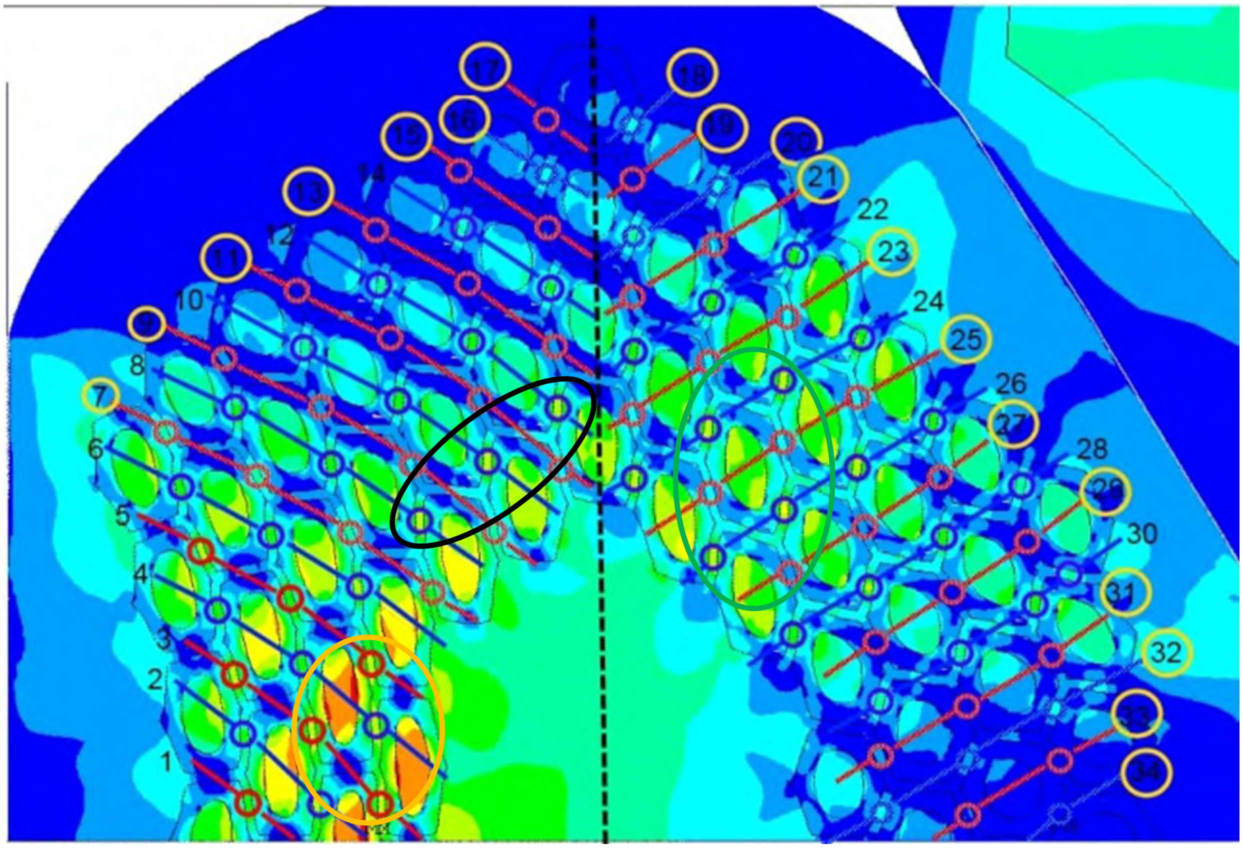


Figure 24 – Strain map in strand one

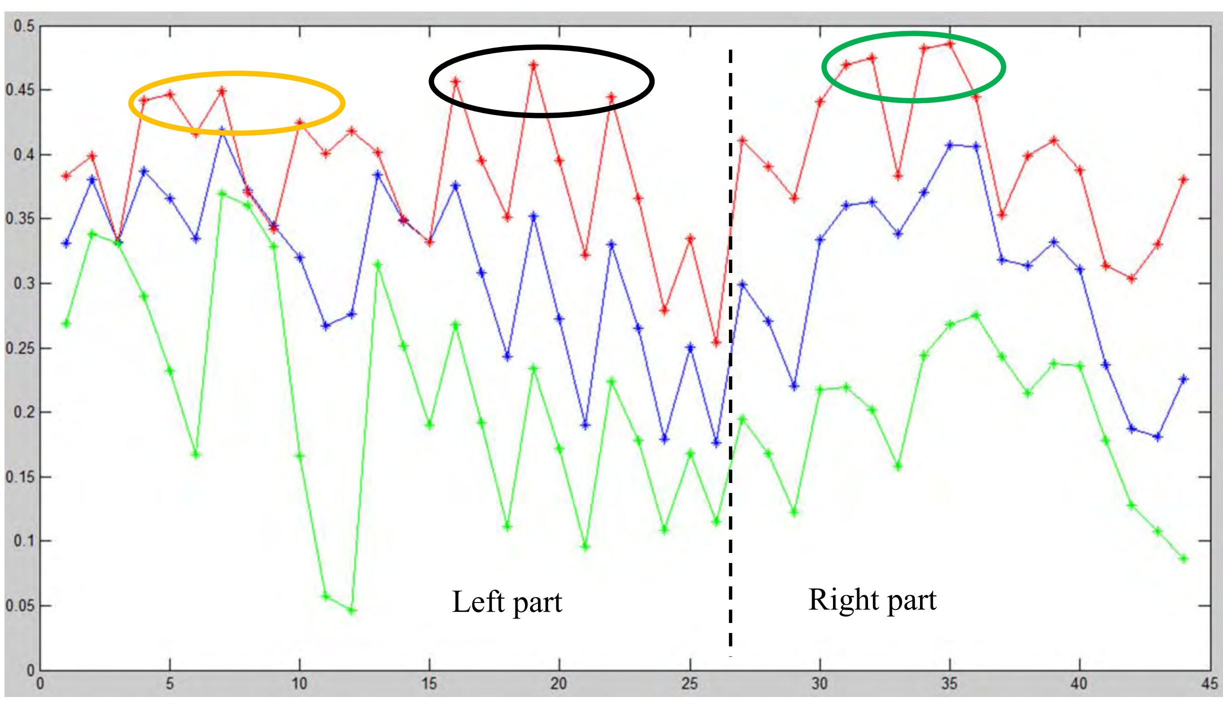


Figure 25 – Strain trend in strand one for $\gamma = -0.3$ (red), $\gamma = -0$ (blue) and $\gamma = 0.5$ (green)

Conclusions

A simplify model of a single strand has been created and his properties set up in order to fit the results found for the deformation of the detailed model. This simplified model has been used to build a whole cable model which an acceptable DOF, so that the simulation wouldn't take too much time. Results obtained from this model have been compared with experimental results (measurement comparison shows a good match).

These analyses let us understand that the rectangular deformation gives most part of plastic energy to strand number one (which is the critical one).

The keystoning deformation extends the areas with high values of plastic energy, but it doesn't really change max value in strand one. Anyway, strand two and three become really interested in deformation, so that they could reach critical conditions.

Damaged subelements in experimental samples are always found in strand number one (for rectangular deformation) and sometimes in strand number two (for keystoning deformation), confirming the model's results.

A study of the effect of the number of strands (N) has been performed; we found out that N is not really important in deformation level in strand one (as long as you give to the cable the same displacement in x direction for all N).

Concentrating on rectangular deformation, analysis on the detailed model of strand one have been performed.

We firstly applied the same load cases actually used to build real cables, so that we could compare results with the pictures obtained with the microscope. The detailed model of the strand detects two critical areas, but only one is damaged in experimental samples. Anyway we dodn't have a large specimen to compare to; Marianne Bossert (the technician who works with the microscope) says that in some cables (made with different load cases) she noticed damages also in the other area detected by the model.

An analysis of roller's x distance has been performed. When the rollers get closer (more intense load for the strand) the max strain in the copper spaces increases. The critical areas move from left part to right part of the strand.

More analysis with different values of rollers' x distance should be run in order to confirm this behavior and a more accurate comparison with experimental results should be performed to test model's accuracy and usefulness in predicting damages in future cables design.