Measurement of magnetic permeability of steel laminations of Booster gradient magnets

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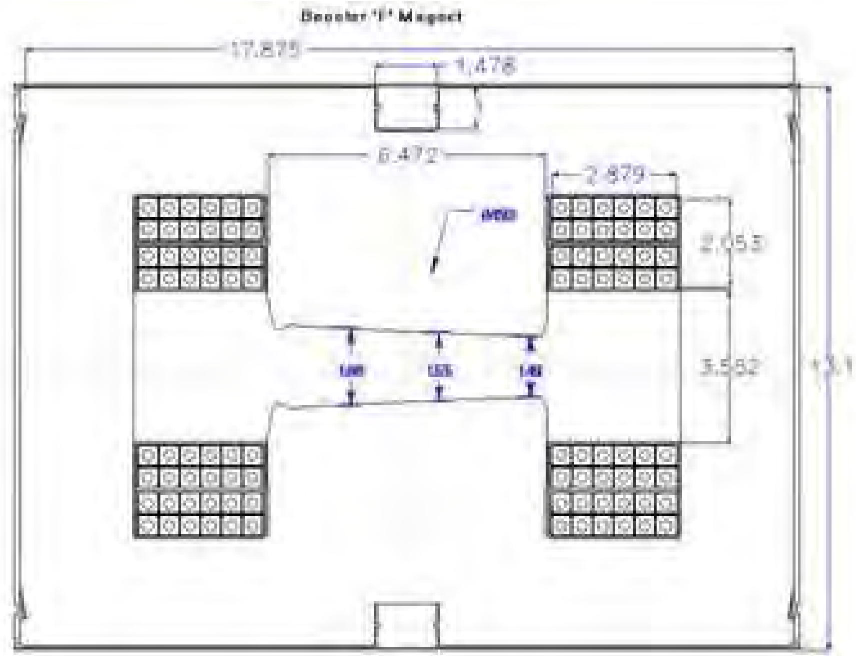
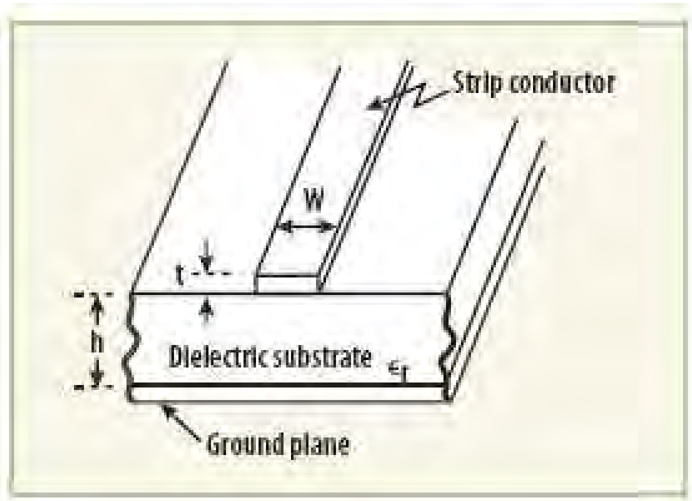
Problem

- Calculation of impedance of Booster gradient magnets
- Unknown magnetic permeability of the steel



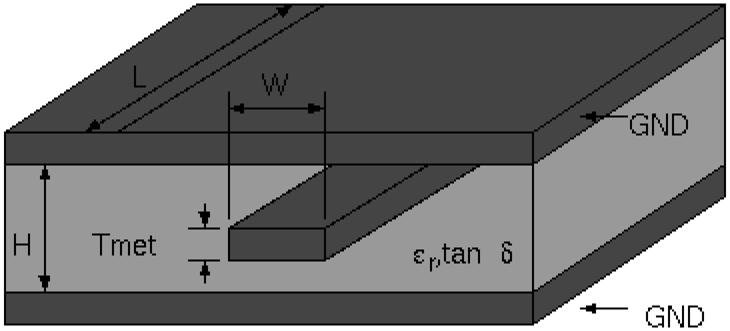
Idea of measurement

Electromagnetic wave propagation in strip lines depends upon properties of materials, including magnetic permeability



Microstrip line

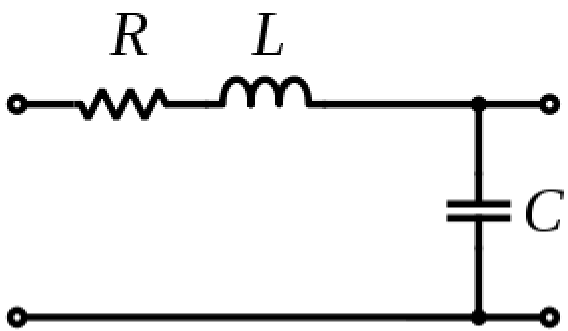
Network analyzer



Strip line

1D transmission line

Basic element of transmission line:



Telegrapher's equations:

$$\begin{cases} \dfrac{\partial U}{\partial x} = -IR - L\dfrac{\partial I}{\partial t} \\ \dfrac{\partial I}{\partial x} = -C\dfrac{\partial U}{\partial t} \end{cases}$$

Harmonic solutions:

$$U = A\exp(i\omega t - ikz) + B\exp(i\omega + ikz)$$

$$I = \frac{A\gamma}{R + i\omega L}\exp(i\omega t - ikz) - \frac{B\gamma}{R + i\omega L}\exp(i\omega t + ikz)$$

$$k^2 = -i\omega C(R + i\omega L)$$

Wave impedance: $\rho = \dfrac{U}{I} = \sqrt{\dfrac{R + i\omega L}{i\omega C}}$

Microstrip line parameters

The simplest formulae (valid if W>>H) for parameters per unit length:

$$C = \varepsilon\varepsilon_0 \frac{W}{H} \qquad L = \mu_0 \frac{H}{W} \qquad R = \frac{(1+i)}{W}\left(sqrt\left(\frac{\omega\mu}{2\sigma}\right)_{strip} + sqrt\left(\frac{\omega\mu}{2\sigma}\right)_{ground} \right)$$

More complicated formulae exist, which take into account edge effects.

$$\varepsilon = \varepsilon' - i\varepsilon''$$

$$\mu(\omega) = \mu'(\omega) - i\mu''(\omega)$$

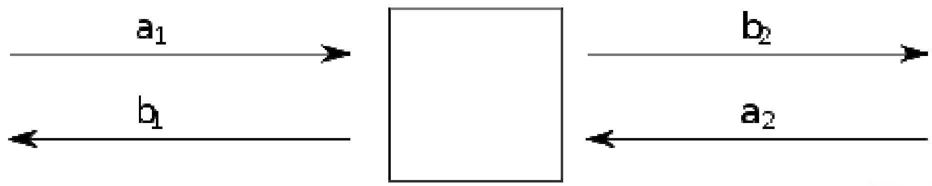
Loss tangent: $\tan\delta = \frac{\varepsilon''}{\varepsilon'}$

If resistive losses are negligible (for example, in the case of copper), then

$$\rho \approx \sqrt{\frac{L}{C}} \approx \rho_0\left(1 + i\frac{\delta}{2}\right) \qquad kl \approx l\omega\sqrt{LC} \approx \omega\tau\left(1 - i\frac{\delta}{2}\right)$$

S-parameters

Definition:

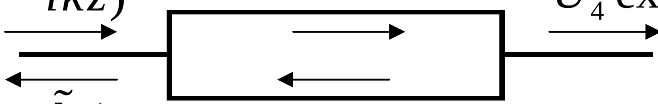


$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

S-parametes are measured by network analyzer

Our case (symmetric):

$U_0 \exp(i\omega t - i\tilde{k}z)$ $U_2 \exp(i\omega t - ikz)$ $U_4 \exp(i\omega t - i\tilde{k}z)$



$U_1 \exp(i\omega t + i\tilde{k}z)$ $U_3 \exp(i\omega t + ikz)$

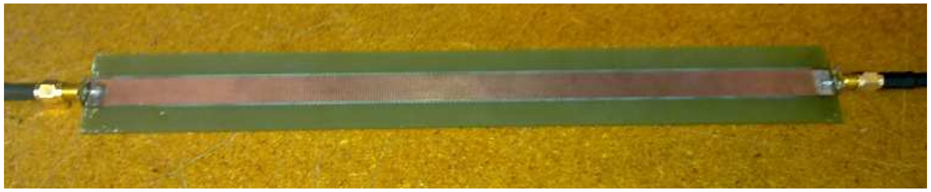
$$S_{11} = \frac{i(\kappa^2+1)\tan kl}{2\kappa + i(\kappa^2+1)\tan kl}$$

$$S_{21} = \frac{2\kappa}{2\kappa \cos kl + i(\kappa^2+1)\sin kl}$$

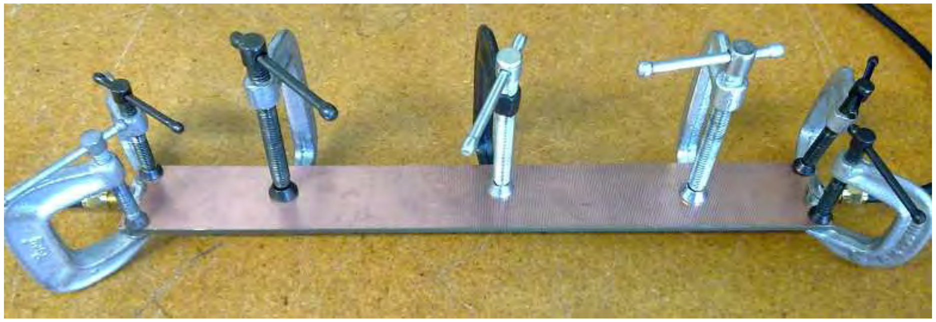
$$\kappa = \frac{\rho}{Z_0}$$

Experimental setup

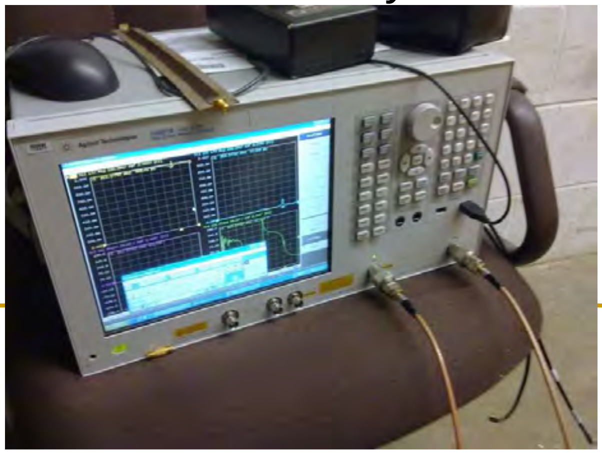
Copper microstrip line



Copper strip line



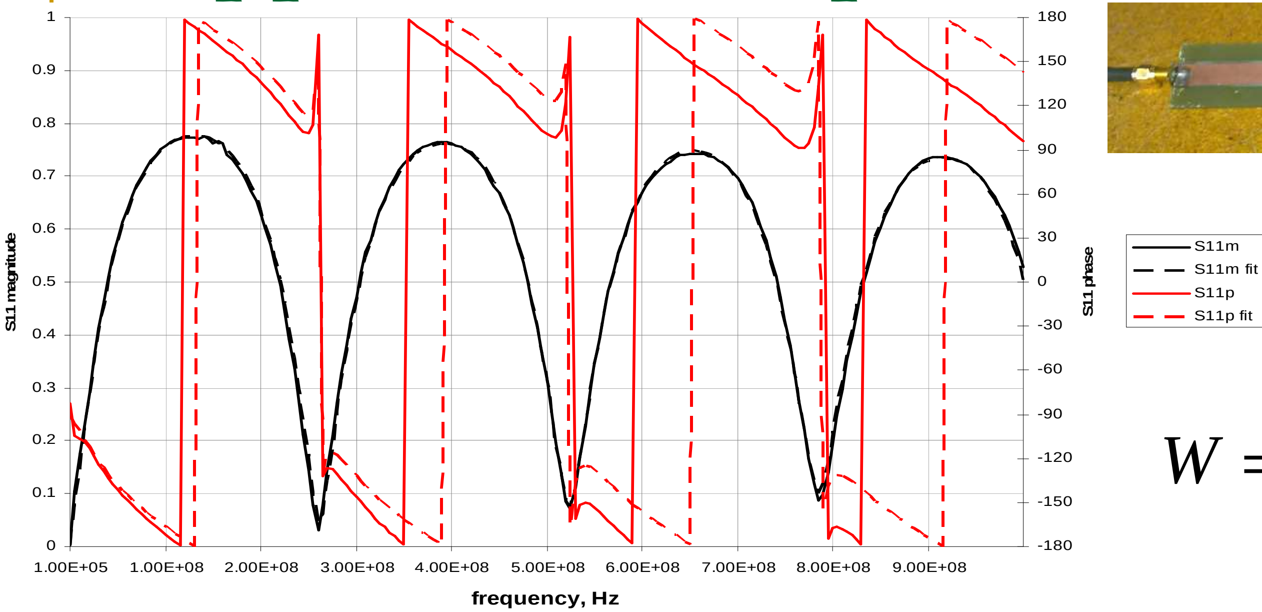
Network analyzer



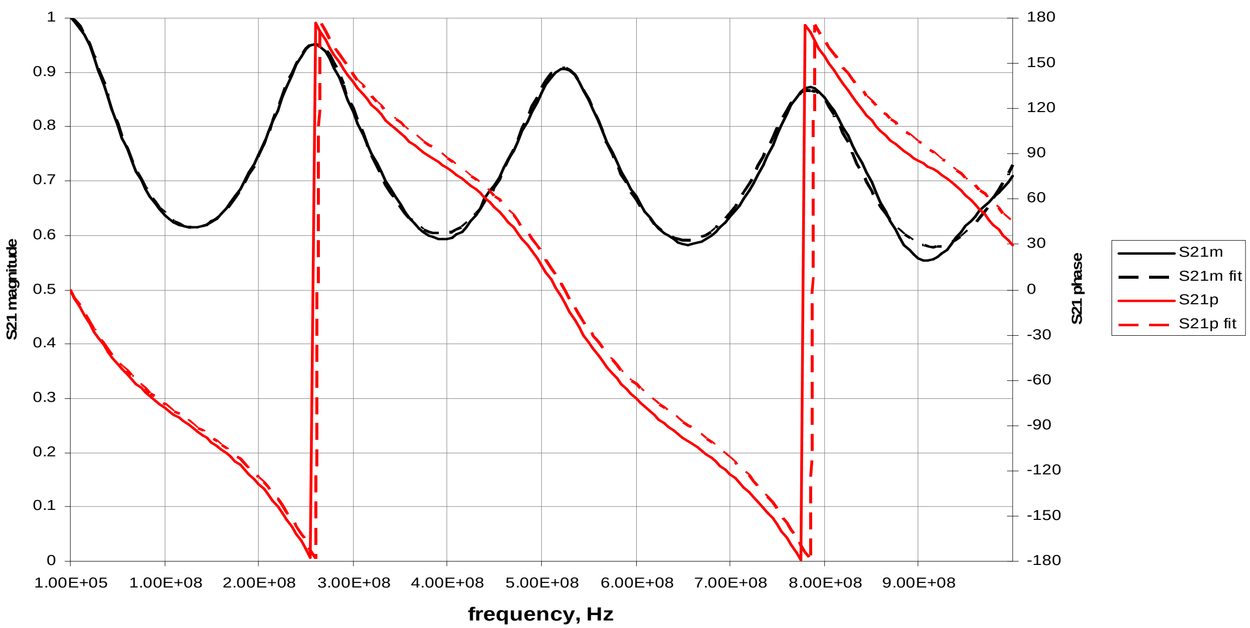
Tapering

Steel microstrip line

Copper microstrip line



$$W = 12mm \quad H = 1.4mm$$

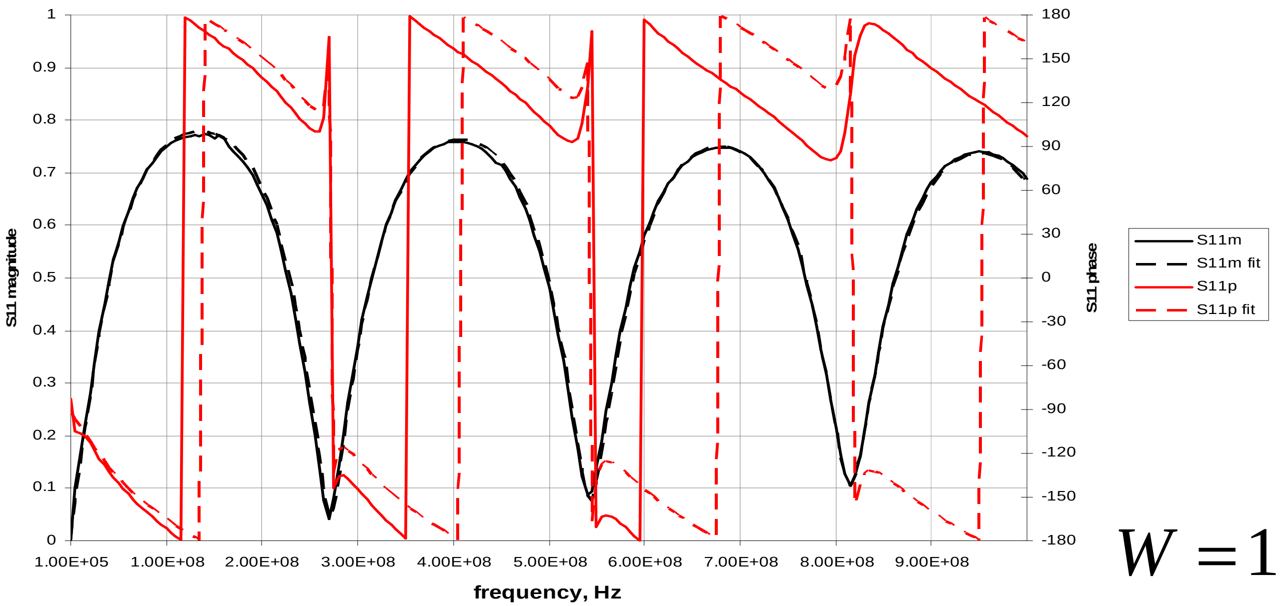


$$\rho_0 = 17.4\Omega$$

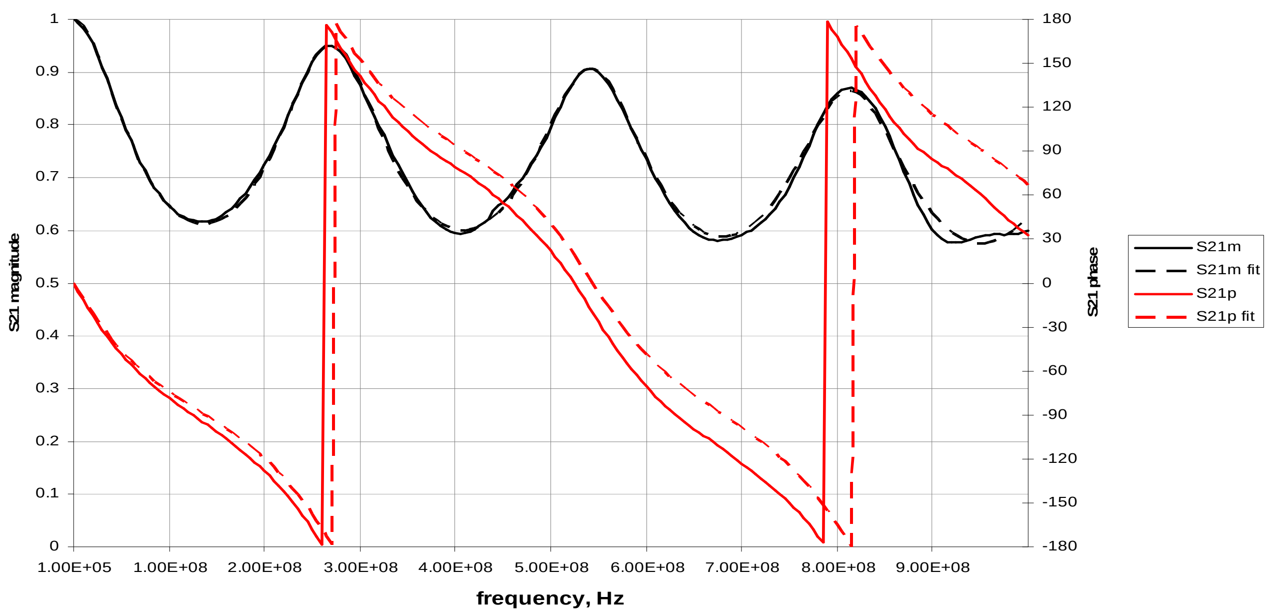
$$\tau = 1.91 \cdot 10^{-9} \, {}^{\text{sec}}\!/_{rad}$$

$$\delta = 0.02$$

Tapered copper microstrip line



$$W = 12mm \quad H = 1.4mm$$

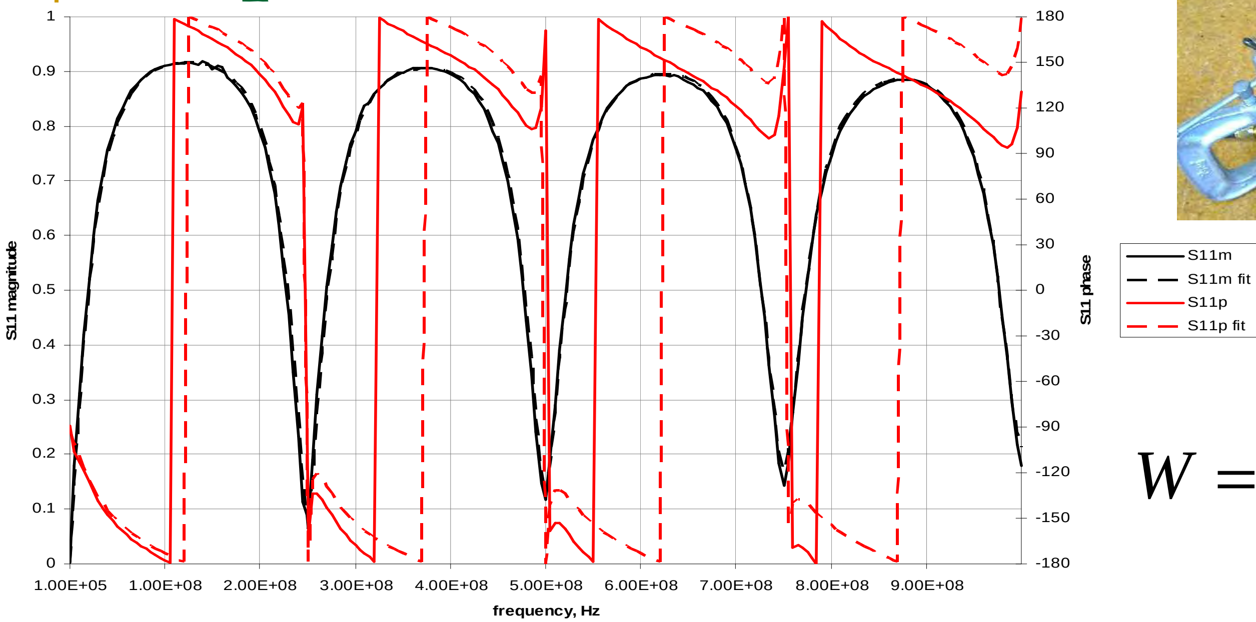


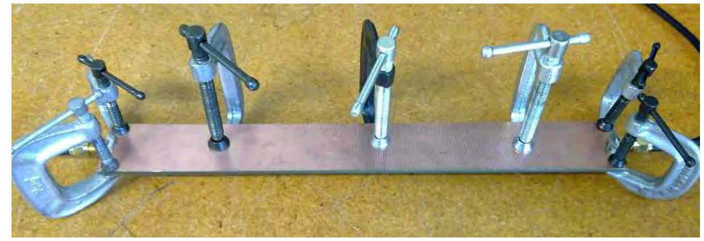
$$\rho_0 = 17.3\Omega$$

$$\tau = 1.84 \cdot 10^{-9} \sec/_{rad}$$

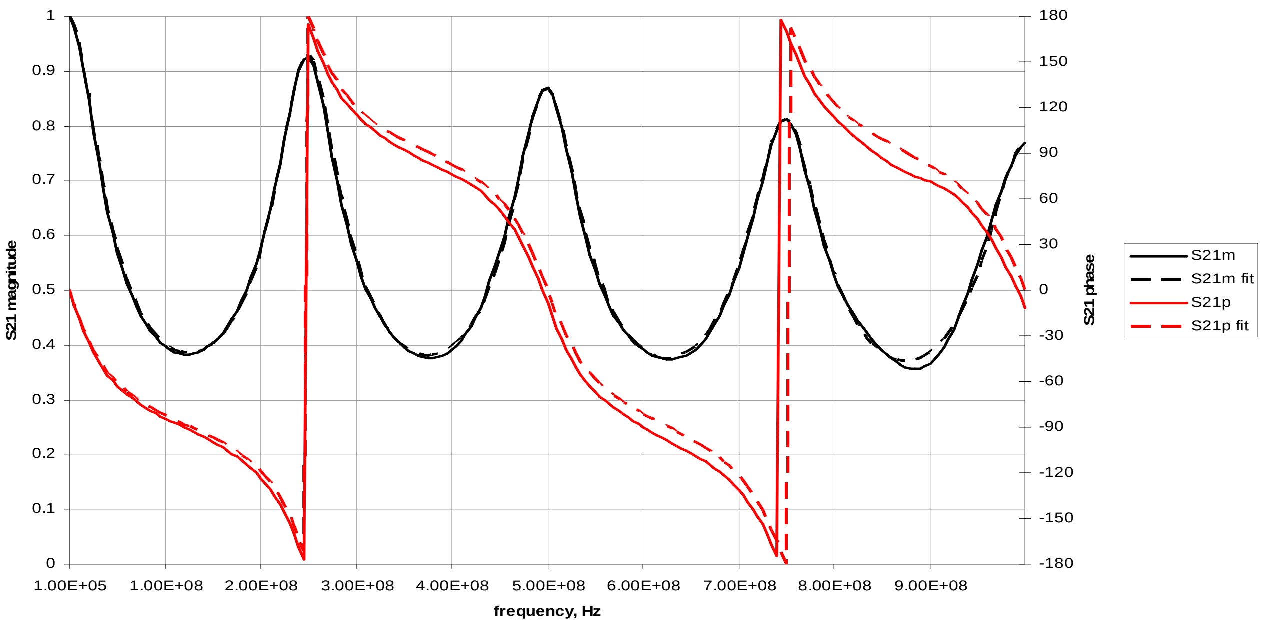
$$\delta = 0.02$$

Strip transmission line





$$W = 12mm \qquad H = 1.4mm$$

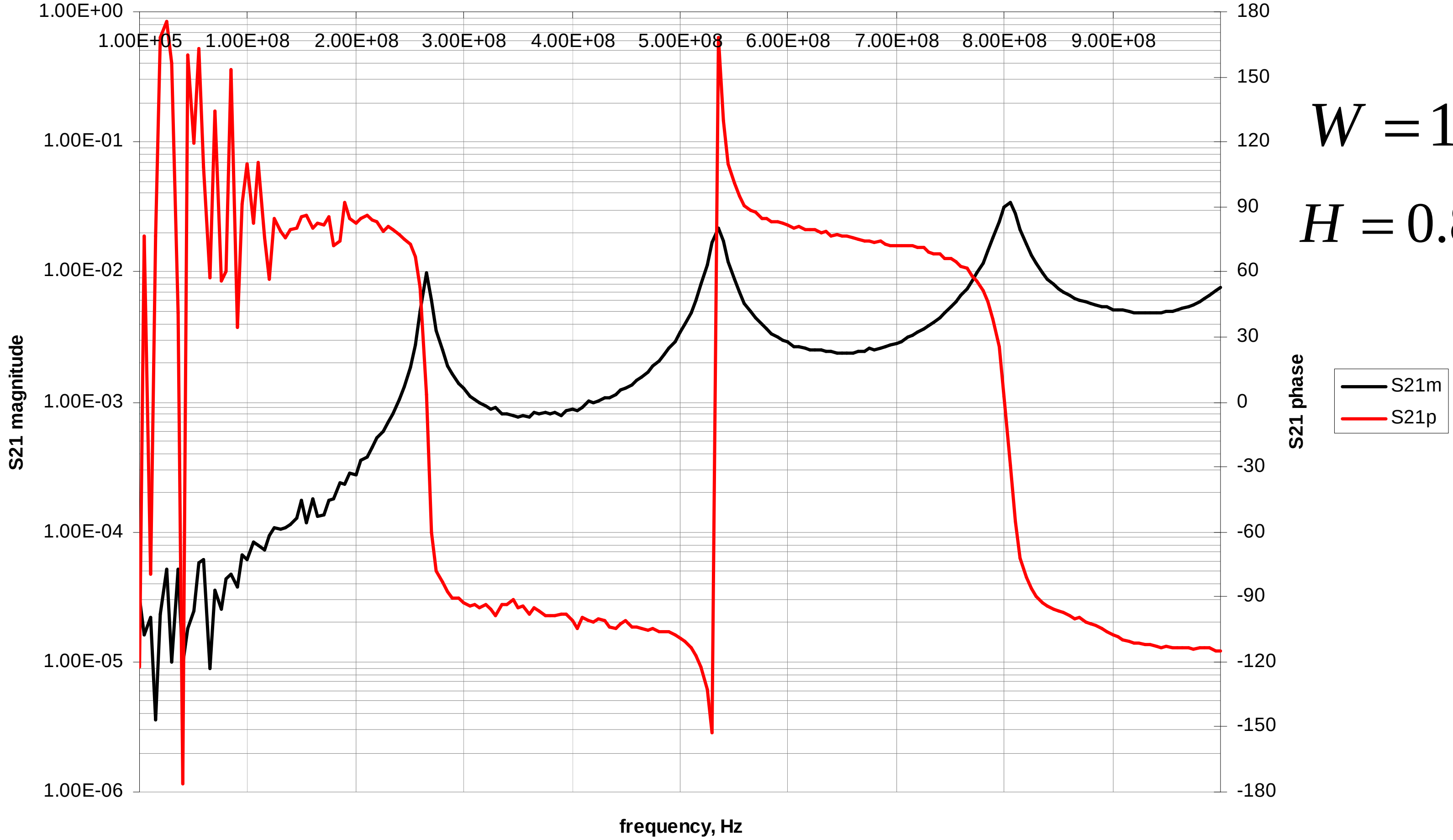


$$\rho_0 = 10.1\Omega$$

$$\tau = 2 \cdot 10^{-9}\ \mathrm{sec}/rad$$

$$\delta = 0.02$$

Weakly-linked resonator

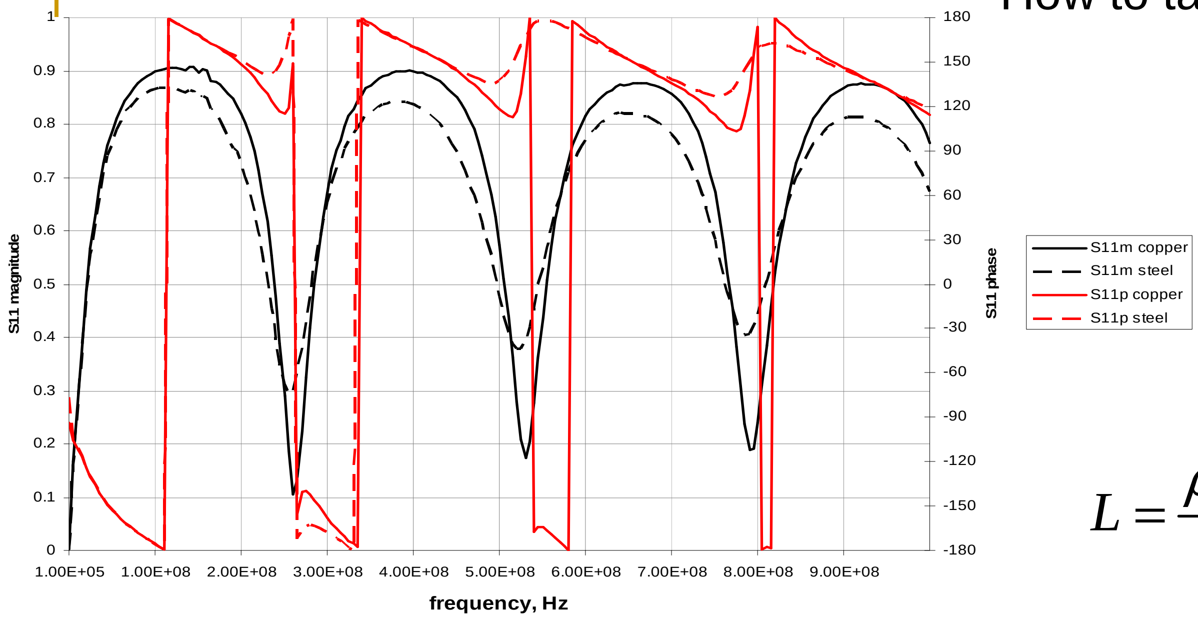


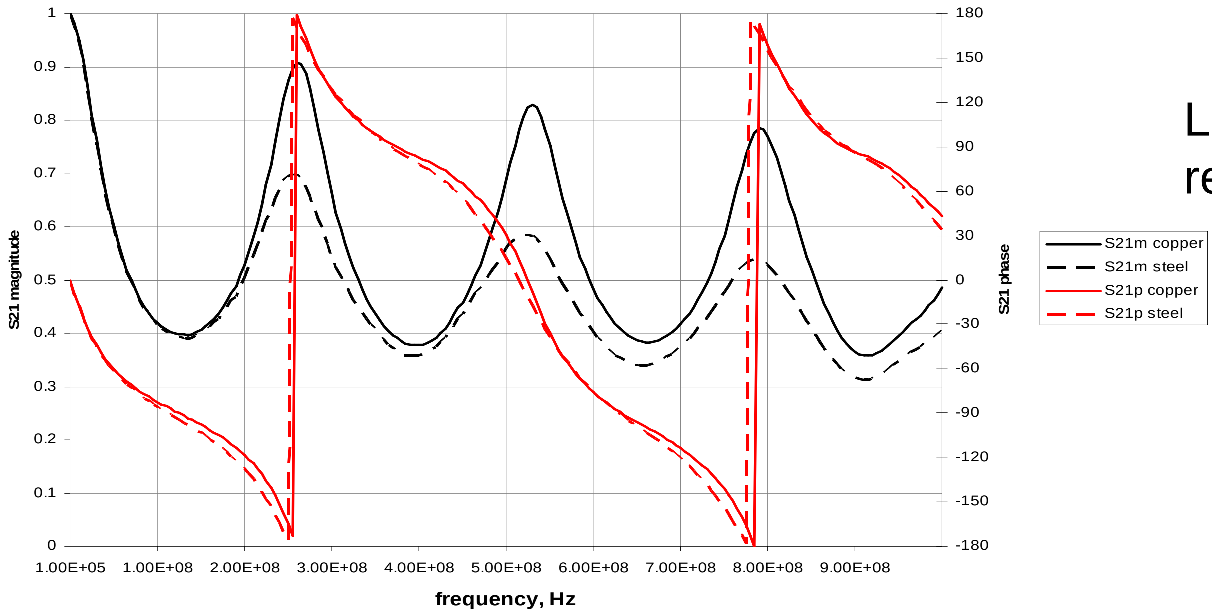
$W = 12mm$

$H = 0.8mm$

$\tau = 1.88 \cdot 10^{-9} \ \mathrm{sec}/rad$ instead of $\tau = 1.85 \cdot 10^{-9} \ \mathrm{sec}/rad$

Steel





How to take into account resistive losses:

$$\tau_s = \tau_c \sqrt{1 + \frac{R}{i\omega L}}$$

$$\rho_s = \rho_c \sqrt{1 + \frac{R}{i\omega L}}$$

$$L = \frac{\rho_c \cdot \tau_c}{l} \qquad R = \frac{(1+i)}{W} sqrt\left(\frac{\omega\mu\mu_0}{2\sigma_s}\right)$$

$$\sigma_s = 2.3 \cdot 10^6 \frac{S}{m}$$

Landau-Lifshitz ferromagnetic resonance model:

$$\mu = 1 + \frac{\mu_s}{1 + i\frac{f}{f_a} - \left(\frac{f}{f_r}\right)^2}$$

Results

- Technique for determining necessary parameters is developed
- Experimental investigation of the problem is carried out
- Rough estimation of magnetic permeability is obtained

Plans

- Solve problem of additional phase shift
- Carry out experiments in strong dc magnetic field (~1-2 T)

Thank you!