Lattice Matching using TRACK

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Goals

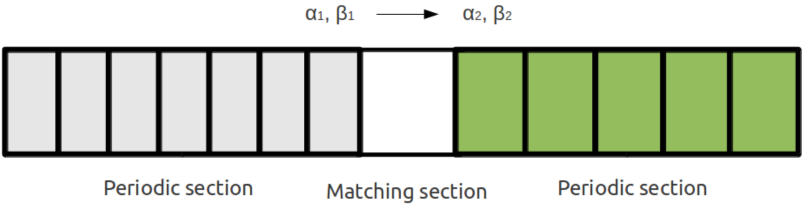
- Write a stand-alone program to do basic matching of Twiss parameters.
- Use TRACK to define lattice functions and to obtain transfer matrices.
- Assess the viability of this approach (is it fast enough to be usable?).

Information

TRACK is a linac tracking code from ANL. As of now, it doesn't have usable matching capabilities.

Matching problem

The beam is matched when its envelope is quasi-periodic.



The full transfer matrix is 6x6. It can be parameterized.

Elements can be used for matching

Transverse

- Solenoids (strength)
- Quadrupoles (strength)
- Drifts (length)

Longitudinal

- RF cavities (field, phase)
- RF gaps (field, phase)

The main idea:

- Construct the objective function.
- Obtain the minimum of the function.
- The choice of weights is **very important** (strongly influences on convergence).

Non-linearity of lattice function (2D)

The matrix of simple FODO cell in 2D case is below

$$\begin{pmatrix} -\frac{L}{f_2} + \left(1 + \frac{L}{f_1}\right)\left(1 - \frac{L}{f_2}\right) & L + L\left(1 + \frac{L}{f_1}\right) \\ -\frac{1}{f_2} + \frac{1-\frac{L}{f_2}}{f_1} & 1 + \frac{L}{f_1} \end{pmatrix}$$

Twiss parameters transformation

$$\begin{pmatrix} \alpha_d \\ \beta_d \\ \gamma_d \end{pmatrix} = \begin{pmatrix} m_{11}^2 & -2m_{11}m_{12} & m_{12}^2 \\ -m_{11}m_{21} & (m_{11}m_{22} + m_{12}m_{21}) & m_{12}m_{22} \\ m_{21}^2 & -2m_{21}m_{22} & m_{22}^2 \end{pmatrix} \begin{pmatrix} \alpha_i \\ \beta_i \\ \gamma_i \end{pmatrix}$$

$$\rho(\alpha_i, \beta_i, \vec{p}) = \frac{1}{w_1}(\alpha_d - \alpha_i)^2 + \frac{1}{w_2}(\beta_d - \beta_i)^2$$

where α_d, β_d are desired values of Twiss parameters, w_1, w_2 - weights.

Non-linearity of lattice function (4D and 6D)

- In 4D case we proceed with 4 quadrupoles and therefore $\alpha_{i_k}(f_1, f_2, f_3, f_4), \beta_{i_k}(f_1, f_2, f_3, f_4)$.
- In 6D we need at least two additional degrees of freedom come from the cavity, so α and β are functions of 6 variables.
- The functions depend on its parameters strongly non-linearly.

Objective function

$$\rho(\alpha_{i_x}, \beta_{i_x}, \alpha_{i_y}, \beta_{i_y}, \alpha_{i_z}, \beta_{i_z}, \vec{p}) = \frac{1}{w_1}(\alpha_{d_x} - \alpha_{i_x})^2 + \frac{1}{w_2}(\beta_{d_x} - \beta_{i_x})^2 + \frac{1}{w_3}(\alpha_{d_y} - \alpha_{i_y})^2 + \frac{1}{w_4}(\beta_{d_y} - \beta_{i_y})^2 + \frac{1}{w_5}(\alpha_{d_z} - \alpha_{i_z})^2 + \frac{1}{w_6}(\beta_{d_z} - \beta_{i_z})^2$$

Minimization algorithm

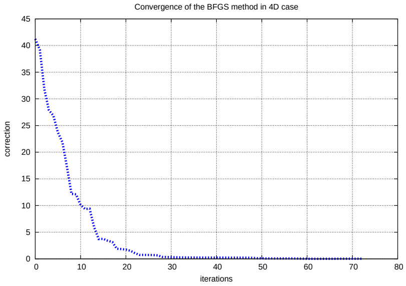
The Broyden-Fletcher-Goldfarb-Shanno (BFGS) method was chosen.

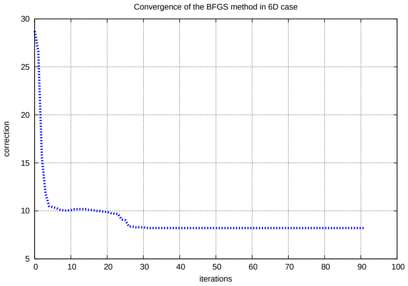
Main properties:

- BFGS is a multidimensional quasi-Newton's method.
- Method approximates the inverse Hessian matrix at every step.
- Only first derivatives are computed.
- Convergence of the method is asymptotically quadratic.

BFGS routines come from GSL.

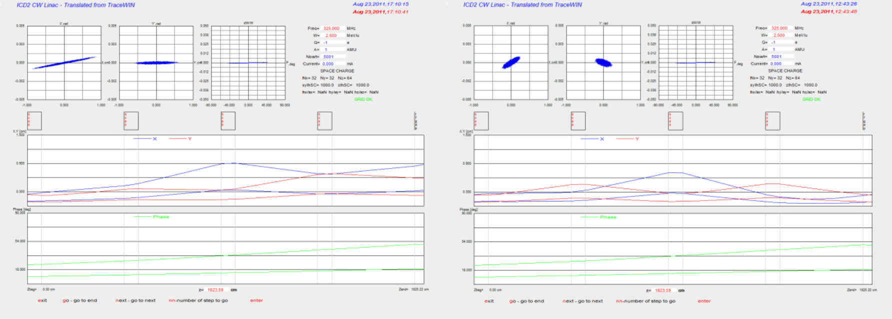
Convergence of the method





Convergence of the method is limited by round-off errors, but it is still greater than linear.

Application example



Mismatched   Matched

	k1	k2	k3	k4
Mismatched	-30.0000	25.0000	-33.0000	27.0000
Matched	-55.0143	45.0046	-55.0143	45.0002
Exact	-55.0000	45.0000	-55.0000	45.0000

Conclusions and future work

Conclusions

- This routine is fast enough to be usable.
- We have found out that matrix output did not have enough significant digits. TRACK was modified accordingly (up to 12 significant digits).
- Tranverse matching works fast and converges perfectly.
- The problem of initial parameters is also very important.

Future work

- Full 6D matching.
- Different types of matching.
- More complicated cases.