Zero model for high currents in turn to turn shorts

CLAS12 project at Technical Division, FermiLab

Giovanni Gabrielli

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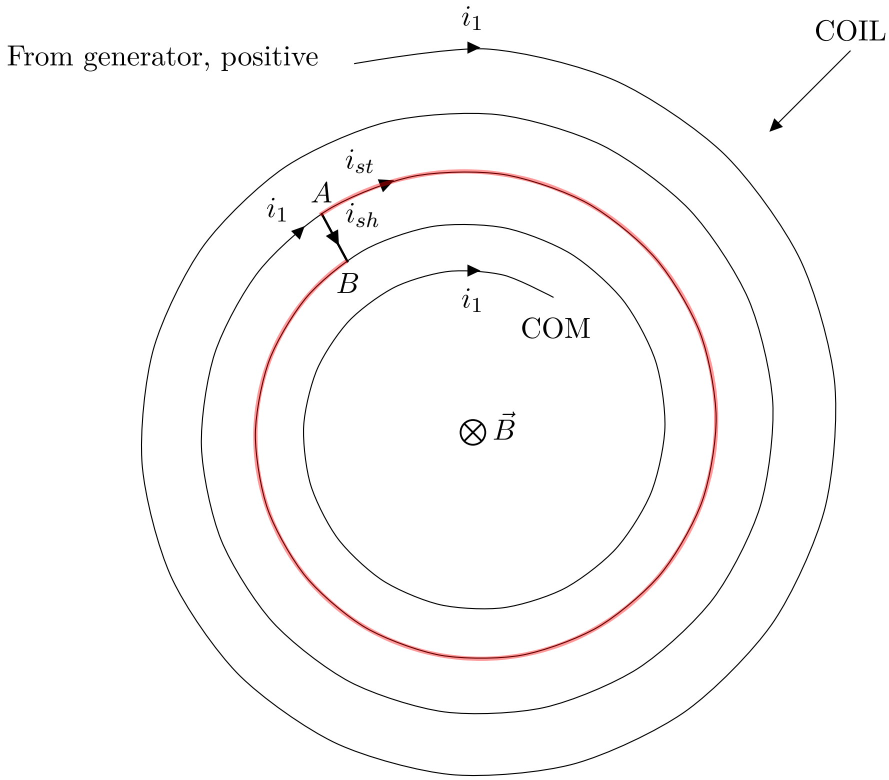
Contents

1 Introduction

1.1 Description of the system

The experiment consists in modeling and understanding the behaviour of voltages and currents travelling in the coil when a short is placed between two consecutive turns. The coil, a double-layered spiral of superconductive material with 17 turns per layer, is connected to a sine wave voltage supply; than we use a scope to measure the total output current and the current travelling the short, consisting in a simple piece of wire. We use a Lock-in amplifier to measure all the voltages and an AC impedance analyzer for the total impedance of the coil, both shorted and not shorted, and for the impedance of the short itself.

The following is a graphic representation of the system.



1.2 Hypotheses

To obtain a rough explanation of the phenomenon, we have to accept a few hypotheses that will help us in the maths and at the same time are not too far from the reality.

1. We are working at a frequency such that the impedance of the coil is almost completely inductive. Thus, we assume that our coil is an ideal inductor whose impedance is $Z = j\omega L$ and the voltage at its ends is $V = j\omega\Phi$, where Φ is the magnetic flux flowing through it. So we neglect any resistive or capacitive parasitic effects, that however could not be neglected at lower or higher frequencies.

2. The coil is composed of N turns that have all the same shape, position and area A. This is not a bad approximation if we are dealing with a huge coil, in which the difference of area between any pair of turn is much less than the area of any single turn. In this case, we can assume that $\Phi = AB_{tot}$, where B_{tot} is the average total magnetic

field generated by the turns of the coil and it is the sum of the single contributes B_n given by each turn.

3. Any turn that is traveled by a current i generates an average magnetic field proportional to that current: that is $B = ki$, being k a proportionality constant equal for all the turns, thanks to the previous hypothesis.

2 Dissertation

2.1 Facts

First of all, we have to notice that the voltage V applied to the coil is always the same, so thanks to hypotesis 1 even the flux Φ has to be the same, both with and without a short. This, thanks to hypotesis 2, allows us to say that the average total magnetic field generated by the turns must be the same, that is

$$B_{tot} = \sum_{n=1}^{N} B_n = constant \tag{1}$$

Then, thanks to hypotesis 3 we can say that

$$\sum_{n=1}^{N} B_n = k \sum_{n=1}^{N} i_n \tag{2}$$

that leads us to the central point, that is

$$\sum_{n=1}^{N} i_n = constant \tag{3}$$

where i_n is the current travelling the n-th turn. Obviously, for a non-shorted coil, this current must be the same for every turn (we use a lumped element model).

On the other hand, through an AC impedance measurement we can clearly notice that, when we put a short between two turns, the impedance of the whole coil lowers in magnitude, frequency being equal. Let's say that it is reduced by a factor $\alpha > 1$. Thus, since we are not changing the voltage, that means that the current feeding the coil is incremented by the same factor.

2.2 Comparing currents

Let's now evaluate equation (3) both with and without shorts. Our goal here is to try and estimate the value of the current running through the short i_{sh}.

Without any short, as already said the current must be the same for every turn, since they are in series, and equal to the total current feeding the coil. Let it be i_0. So we have

$$\sum_{n=1}^{N} i_n = \sum_{n=1}^{N} i_0 = N i_0 \tag{4}$$

On the other hand, when we put a short between two turns, basically one turn is excluded from the series, and through the short it makes a loop closed on itself (except for the short, that is in common with the main current path, but it has a small impedance). So, in this loop, that is the shorted turn, current is free to run and it can be completely different from the one running in the other turns. Let this be i_{st}, where *st* stands for "shorted turn" - notice that this is not i_{sh}, which is running through the short itself. Finally, let the current feeding the whole coil, the same that is travelling every other turn, be i_1. In this case, equation (3) is nothing but

$$\sum_{n=1}^{N} i_n = (N-1)i_1 + i_{st} \tag{5}$$

Then, the last two equations put together give us

$$N i_0 = (N-1)i_1 + i_{st} \tag{6}$$

From the schematic of the system it is evident that

$$i_{sh} = i_1 - i_{st} \tag{7}$$

while we already said that

$$i_1 = \alpha\ i_0 \tag{8}$$

Now, by replacing (8) and (7) into (6) we get

$$N i_0 = N i_1 - (i_1 - i_{st}) = N \alpha i_0 - i_{sh} \tag{9}$$

that gives us an estimation of the current in the short:

$$\boxed{i_{sh} = N(\alpha - 1) i_0} \tag{10}$$

From this formula we can understand that the current in the short will be even much, much higher than the current usually running through the non-shorted coil. One more thing is worth to be noticed, and that is the current in the shorted turn i_{st}: from (6) and (8) we have

$$i_{st} = [N(1-\alpha) + \alpha]\, i_0 < 0 \tag{11}$$

if N is high enough. This means that in that turn the current has changed its sign.

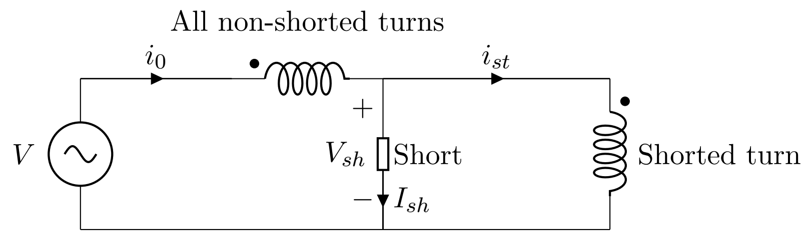
2.3 Voltages

There's one more thing to understand. Let's consider the shorted turn and the voltage drop between the ends the short. Let A be the end of the short that is nearest to the positive output of the generator, and let B the other one. Obviously, without a short we had a positive voltage drop V_{AB}, and even with the short it is lower, but still positive. Now, if we follow the turn from A to B, in the first case we had a positive current and a positive voltage drop, while in the second case (with the short) we have a negative current but still a positive voltage.

To explain this apparent paradox, we have to think about what creates the currents in this system: obviously, i_1 is produced by the generator, but I_{st} is not - it runs in a circuit, the shorted turn, that is almost completely independent from the main ciol turns. At least, it is *electrically* independent, but not magnetically: indeed the current i_1 induces a magnetic field, which induces a current in the shorted turn. Since this is an induced current, the electromotive force of that turn will be given by Faraday's Law of induction,

$$\mathcal{E} = -j\omega\Phi \tag{12}$$

where the sign represents Lenz's Law: this means that although the current has changed its sign, it produces a negative voltage - that is, it generates power - thanks to magnetic induction, so actually the voltage changes its sign twice and stays positive. In this situation, the shorted turn switches from being part of the load - together with all the coil's turns - to being the secondary of a transformer, that thus generates power itself, closed on the same load that is the short. This schematic will help understand this situation.



3 Experimental results

After a few measurements, this model appears convincing as it can predict the amount of current flowing in the short: so, knowing the voltage drop across it, it is an indirect measure of the impedance of the short itself - at least a rough approximation of its order of magnitude, given the very low accuracy of this simple method. In the experiment, a 34 turns coil was powered with a 200 kHz, 10 V amplitude sine wave. This frequency has been chosen because it belongs to a range of frequencies where the coil has the most inductive impedance.

Here are the results of the measurements:

1. Non-shorted coil impedance: $555\ \Omega, 78°$ at $200kHz$
2. Non-shorted coil current i_0: $20\ mA$
3. Shorted coil impedance: $258\ \Omega, 77°$ at the same frequency
4. Shorted coil current i_1: $40\ mA$
5. Current in the short i_{sh}: $670\ mA$
6. Minimum turn to turn voltage (that means where the short is) V_{sh}: $80\ mV$
7. Short impedance (piece of wire) Z_{sh}: $162,5\ m\Omega, 86°$ at the same frequency

First of all, we have to notice that even these measurement have a significant amount of error, since 5., 6. and 7. are not perfectly coherent: 5. and 6. actually give an impedance of 119 $m\Omega$, that has a 30% error.

However, let's calculate the predicted amount of current travelling the short, as if we could not measure it - so it will be in the real case.

First of all, the AC impedance and the coil current measurements give us an α factor of around 2. Back to the formulas, and knowing $N = 34$, we easily find from equation (10)

$$i_{sh} = N(\alpha - 1)i_0 = 34(2 - 1)20\ mA = 680\ mA \tag{13}$$

that is a very good approximation of the value we had.

In conclusion, our model is a good approximation of the real system and, above all, gives a theoretical explanation of the phenomenon. This high current in the short, even if it has a very small impedance, is the reason why we still see a significant amount of voltage drop across it: with a small impedance we could expect a small voltage drop, but this doesn't happen because of the very high current that is induced in it.