SRF LINAC Simulation

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Abstract

Simulating the acceleration of protons traveling through the field of a superconducting radio frequency (SRF) cavity to determine the number of cavities required for acceleration is quite complex. This paper describes a simplified model based on the actual PIP – II linac design. It does not take into account transverse RF field and particle dynamics. This means that only the component of the field along the axis of propagation of the particle is considered in the simulation. The simulation describes the exchange of energy between the cavity fields and a single particle as it passes through the cavities. I wrote a program to perform a numerical integration on the cavity field to determine the NTTF curve which shows the energy gained by the particle for any particular β. The number of cavities for acceleration is calculated from the relationship between the Kinetic energy of the beam and the NTTF curve. Even though transverse RF fields are not considered, the result of the simulation is within a reasonable range of the real number of cavities used for the acceleration.

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1. Introduction

This simulation investigates and describes how protons are accelerated in a linear accelerator that uses superconducting radio frequency (SRF) cavities. Specifically, the exchange of energy between the cavity longitudinal electric field and a single charged particle as it passes through the cavities. The simulation also determines the number of cavities required to accelerate the particle from a given energy to a higher energy. Transverse particle dynamics is not considered, which means that only the component of the electric field at the center of the cavity and along the axis of propagation of the particles beam (E_z) is considered. Other components of the field ($E_x, E_y, H_x and\ H_y$) are not considered in this model.

The goal is to accelerate the protons from 2.1 MeV to 1 GeV using five cavity types having different E_z field on axis. In doing this, the phase at which the particle beam gains the maximum energy is first determined. Given the best phase for acceleration, the energy gain for each β and the optimum β (β_{opt}) for each cavity field is determined. The last step that will be described is how the result of the previous calculations is used to determine the number of cavities required to accelerate the particles.

2.1 Overview: Particle Accelerator

A particle accelerator is a machine that uses electromagnetic fields to propel charged particles (subatomic particles, such as, electrons or protons) to nearly the speed of light and to contain them in well-defined beams. Particle accelerators are used to study the origins of our universe, investigate the subatomic structure of the world around us and advance research in medicine, environmental clean-up and more. A particle accelerator can either have an electrostatic field (constant electric field) or an oscillating field (varying electromagnetic field).

There are two types of oscillating field accelerators: linear accelerators (linac) and circular accelerators. The linac propel particles along a straight beam line while circular accelerators propel particles around a circular orbit.

2.2 Overview: Superconducting Radio Frequency (SRF) Cavities.

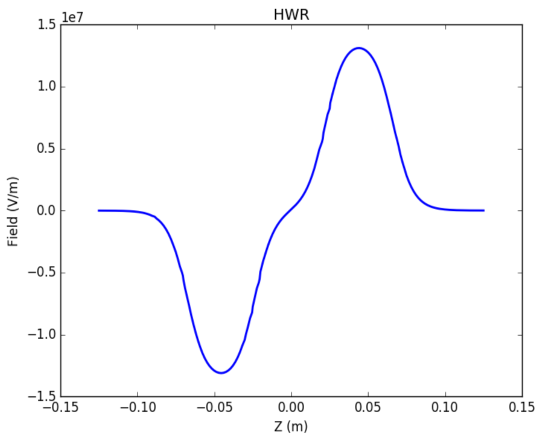
Electromagnetic fields are excited in the SRF cavity by coupling in an RF source with an antenna. Like normal conducting (NC) cavities, SRF cavities are used for accelerating particle beams. The main difference being that SRF cavities are made with a superconducting material. In the past two decades, accelerator facilities have increasingly found superconducting cavities to be more suitable (or necessary) for their accelerators than normal-conducting copper versions. The motivation for using superconductors in RF cavities is not to straightforwardly achieve a net power consumption savings [1].

Even though the electrical resistance is very small in SRF cavities (five orders of magnitude less than NC cavities), the net power consumed is still relatively close to the NC cavities. This is because maintaining the low temperature required for the SRF to work consumes a high amount of power. An advantage for using the SRF cavities is that nearly all RF power goes to the beam. So, the RF source driving the cavity need only provide the RF power that is absorbed by the particle beam being accelerated [1]. A considerable amount of power is lost to the cavity walls in NC cavities, most especially when they are used to accelerate the particles to very high energies. SRF cavities also allow the excitation of high electromagnetic fields at high duty cycle, or even cw, which would otherwise not be achievable using copper in a NC cavity.

3. Simulation

3.1 Cavity Fields

Five cavity types are used in this simplified model of linear accelerator: one 162.5 MHz half wave resonator (HWR (2.1 – 10 MeV)), two different spoke resonators at 325 MHz (SSR1 (10 – 30 MeV) and SSR2 (30 – 180 MeV)) and two families of 650 MHz 5-cell cavities (650 LB (180 – 490 MeV) and 650 HB (0.49 – 1 GeV)). The graphs below represent the electric field at the center of the cavity along the axis of propagation of the particles (z-axis) for each cavity type.



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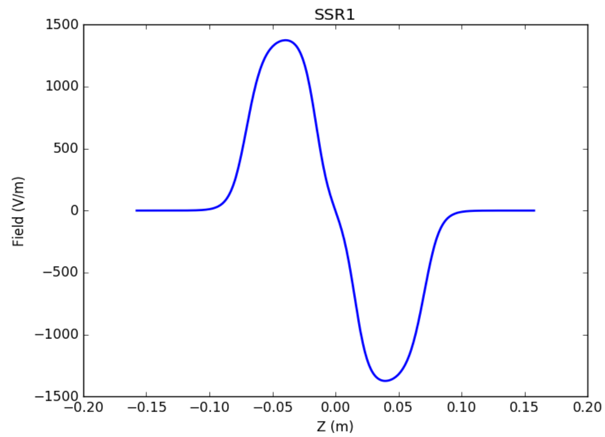

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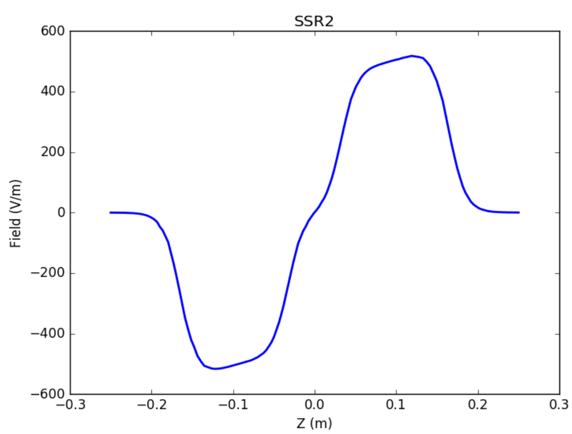

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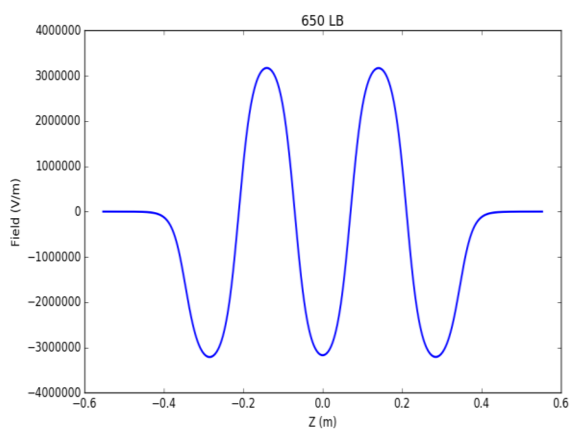

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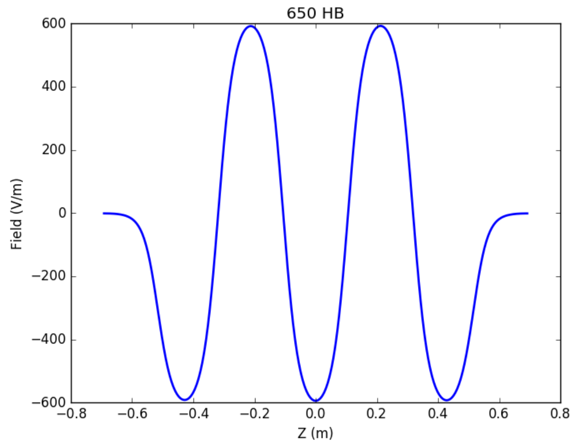


(e)

Figure 1: Cavity types; (a) *HWR (2.1 to 10 MeV) (b) SSR1 (10 to 30 MeV) (c) SSR2 (30 to 180 MeV) (d) 650 LB (180 to 490 MeV) (e) 650 HB (490 MeV to 1 GeV)*

3.2 Energy Exchange (Gain) Profile

The energy gained by a particle as it passes through the cavity is given by the formula below:

$$\Delta W_z = \left| \int_{-l/2}^{l/2} q * E_z(r = 0, z) * e^{i\omega t} dz \right|$$

The integral above takes into account the oscillation of the electromagnetic field with time measured along the z axis and zero transverse offset. To calculate the integral above with respect to the location of the beam (z), it has been assumed that the particle β is constant through the cavity. The assumption allows to rewrite the integral as a function of z instead of time. The time dependence can be converted to position dependence using the equations below.

$$\omega = 2\pi f \quad , c = \boldsymbol{\lambda} f \quad , k = \frac{2\pi f}{c} \quad and\ \beta = \frac{v}{c}$$

$$\Delta W_z = \left| e \int_0^l E_z(r=0,z) * e^{\frac{ikz}{\beta}} dz \right| = \left| e \int_0^l E_z(r=0,z) * (\cos(kz/\beta) + i\sin(kz/\beta))\, dz \right|$$

Where l is the length of the cavity, E_z is the cavity field, ω is the angular frequency, $\boldsymbol{\lambda}$ is the wavelength, v is the particle speed, c is the speed of light, f is the electromagnetic field frequency and k is the wavenumber.

3.2.1 The Function (cos or sin) that Produces the Maximum Energy Exchange

The formulas below are equivalent to the exponential function above, once the proper phase has been chosen and the proper symmetry for the field profile has been noted (phase = 0 or π in this case given that the absolute value is calculated).

$$\Delta W_z = \left| e \int_0^l E_z(r=0,z) * cos(kz/\beta)\, dz \right| \quad or\ \Delta W_z = \left| e \int_0^l E_z(r=0,z) * sin(kz/\beta)\, dz \right|$$

Using the SSR1 field (10 to 30 MeV), after computing the expression within the absolute value sign for the two functions, two examples (for β = 0.1 and β = 0.3) of the resulting field to be integrated to determine ΔW_z are shown below.

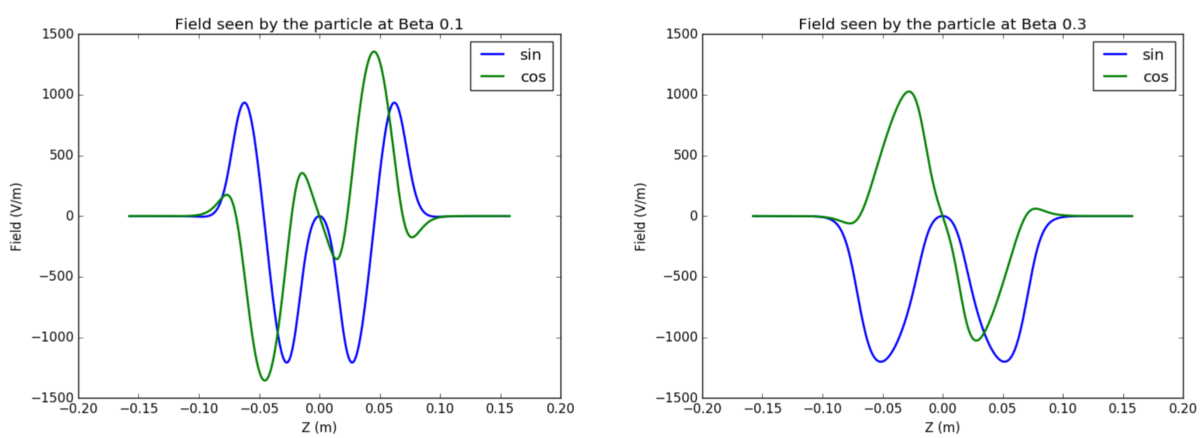


Figure 2: *The fields seen by particle beam at β = 0.1 (left) and β = 0.3 (right)*

The resulting fields were integrated to determine the energy exchange (ΔW_z) not only for the two particle βs in Figure 2, but also for all the particle βs in the given energy range. Then, each ΔW_z was plotted against its corresponding β for all the βs which gives the energy exchange profile. The integration was performed numerically using the trapezoidal rule for integration. The trapezoidal rule works by approximating the region under the graph of a function as a trapezoid and calculating its area. The area under the graph is approximated by parallel trapezoids, the areas of the trapezoids are calculated and summed up to give the definite integral value. The approximation is given by the equation:

$$\int_a^b f(x)dx \approx \frac{1}{2}\sum_{k=1}^{N} h_k * \left(f(x_{k+1}) + f(x_k)\right)$$

Where 'N' is the number of trapezoids and $h_k = x_{k+1} - x_k$

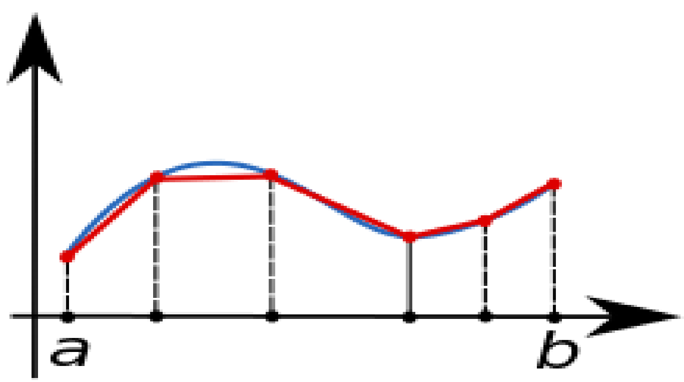


Figure 3: *Example of an integral approximated with trapezoids*

It follows that

$$\Delta W_z = \left|\frac{e}{2}\sum_{k=1}^{N} h_k * \left(f(z_{k+1}) + f(z_k)\right)\right|$$

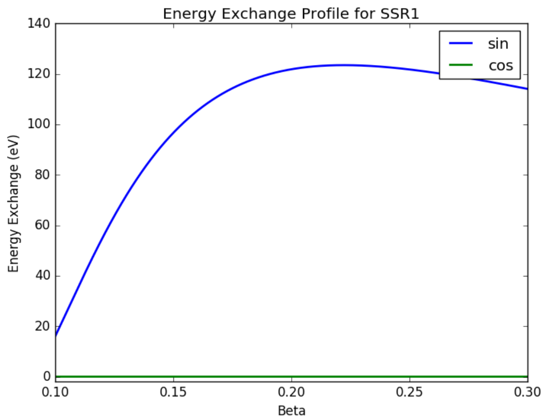
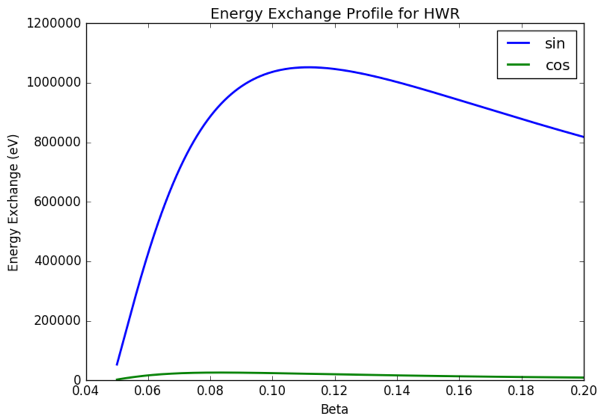


Figure 4.1: *The energy exchange profile for the sine and cosine function for the SSR1 (10 -30 MeV)*

According to the Figure 4.1, for any given β, ΔW_z for the sine function is greater than the ΔW_z for the corresponding cosine function. This shows that for this particular cavity, the sine function produces the higher energy exchange.

The same process is repeated for the other cavity types. The graphs generated for those cavities are shown below.



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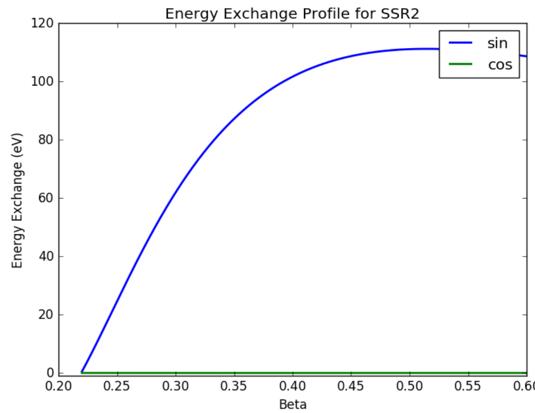

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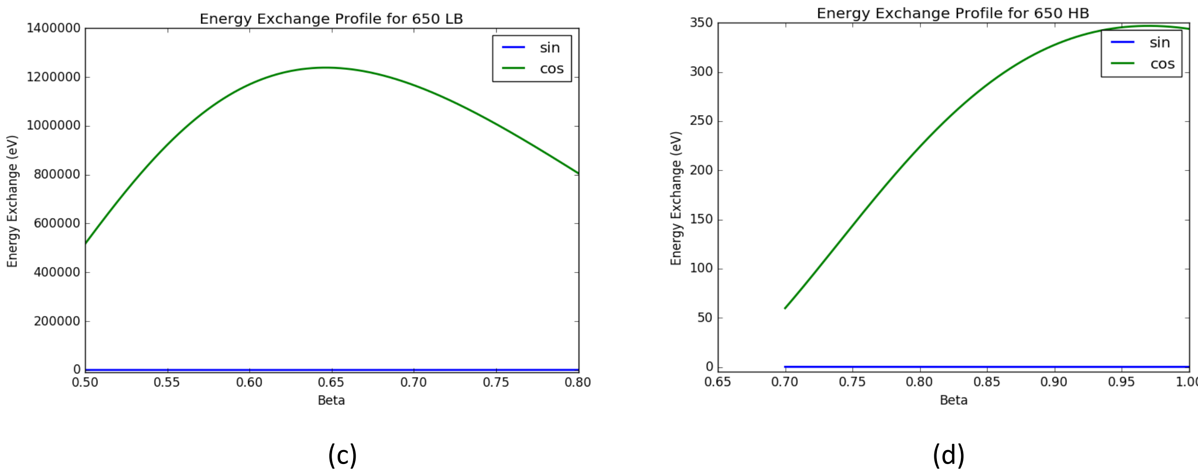


Figure 4.2: *The energy exchange profile for the sine and cosine functions for the* (a) *HWR (2.1 to 10 MeV) (b) SSR1 (10 to 30 MeV) (c) SSR2 (30 to 180 MeV) (d) 650 LB (180 to 490 MeV) (e) 650 HB (490 MeV to 1 GeV)*

The table below summarizes the result for all the cavity types

Cavity Type	Suitable Function
HWR (2.1 – 10 MeV)	Sine
SSR1 (10 - 30 MeV)	Sine
SSR2 (30 – 180 MeV)	Sine
650 LB (180 – 490 MeV)	Cosine
650 HB (490 MeV – 1 GeV)	Cosine

Table 1: *The Cavity types and their corresponding functions that produce the maximum energy exchange*

3.2.2 The Phase for Maximum, Minimum and Zero Acceleration

The result on Table 1 is based on the assumption that the maximum energy exchange occurs when the phase is 0 or π rads for a sine or a cosine function. To determine the actual phase for maximum energy exchange for these fields, the same integral above was computed but in this case, the phase was varied from 0 to 2π.

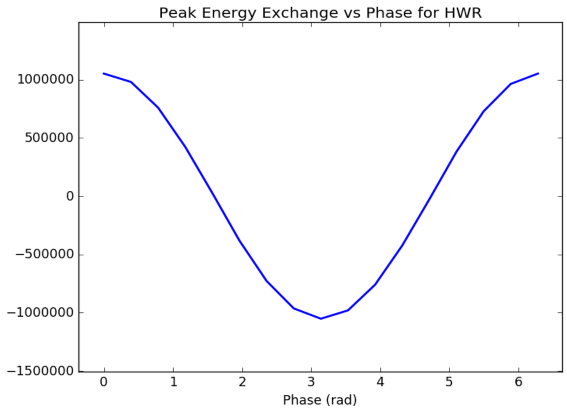
$$\Delta W_z = e\int_0^l E_z(r=0,z) * cos(\left(\frac{kz}{\beta}\right) + \varphi\ ))\ dz$$

$$\Delta W_z = e\int_0^l E_z(r=0,z) * sin(\left(\frac{kz}{\beta}\right) + \varphi\ ))\ dz\,, \qquad where\ \varphi = phase$$

Cavity Type	Phase (rad)			
	Suitable Function	Max Acceleration	Min Acceleration	≈ Zero Acceleration
HWR (2.1 – 10 MeV)	Sine	0	π	$\pi/2, 3\pi/2$
SSR1 (10 - 30 MeV)	Sine	π	0	$\pi/2, 3\pi/2$
SSR2 (30 – 180 MeV)	Sine	0	π	$\pi/2, 3\pi/2$
650 LB (180 – 490 MeV)	Cosine	π	0	$\pi/2, 3\pi/2$
650 HB (490 MeV – 1 GeV)	Cosine	π	0	$\pi/2, 3\pi/2$

Table 2: *The phase for maximum, minimum and zero acceleration for each cavity type..*

For a better visual representation of the result, the Peak Energy Exchange vs Phase was plotted for each field.



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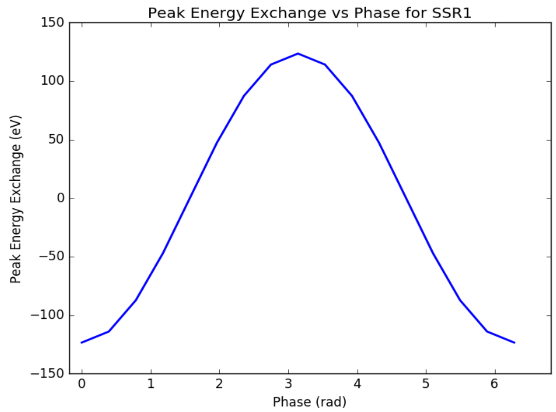

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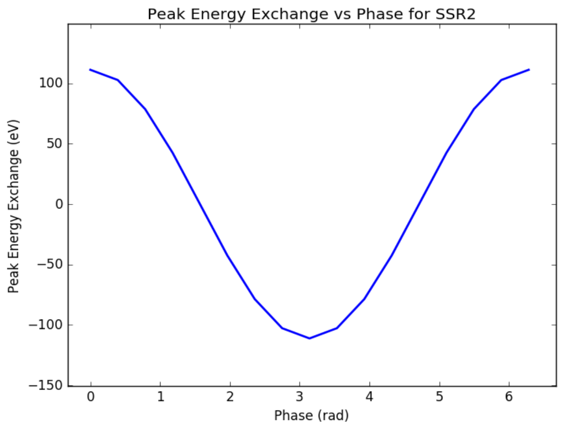

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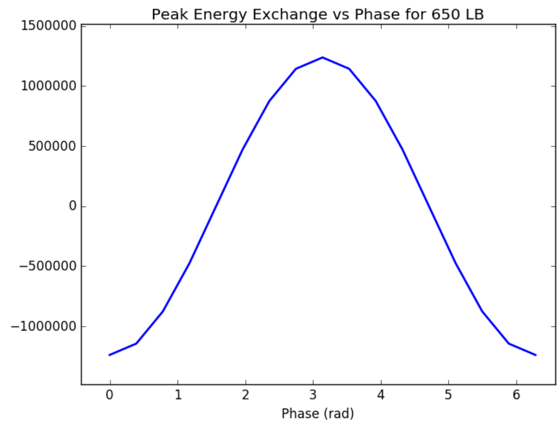

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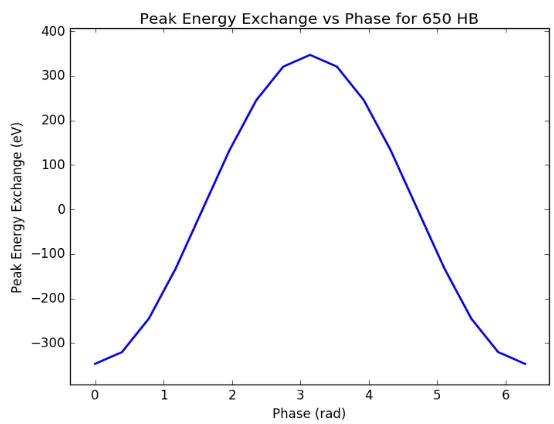


(e)

Figure 5: *Peak energy exchange vs Phase for the* (a) *HWR (2.1 to 10 MeV) (b) SSR1 (10 to 30 MeV) (c) SSR2 (30 to 180 MeV) (d) 650 LB (180 to 490 MeV) (e) 650 HB (490 MeV to 1 GeV)*

The results in Table 2 is evident in Figures 5. And they all agree with the earlier assumption that the maximum energy is transferred at either 0 or π rads. So, the Energy Exchange profiles in Figure 4.1, 4.2 and 4.3 are correct and represent the energy gained by the particle for maximum acceleration for the respective cavity types.

3.3 Normalized Transit-Time Factor (NTTF)

The maximum energy exchange (ΔW_{max}) for a cavity is a preset value which may be different from the estimated values in the Energy Exchange profiles in Figure 4.1, 4.2 and 4.3. In order to match the Energy Exchange profile to the given ΔW_{max}, the energy profile is rescaled for use. To make this process easier and general, a normalized energy exchange profile called the normalized transit time factor is used. The transit-time factor is a factor that takes into account the time variation of the field during particle transit through the gap. It represents a relationship between the energy gained using DC power and RF power (varying field).

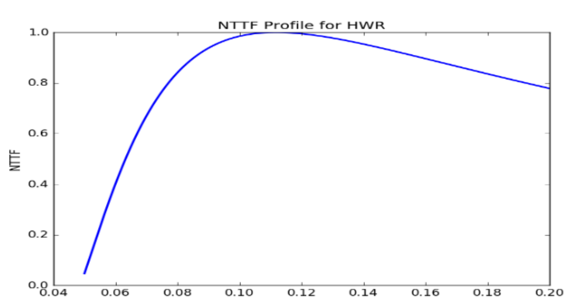
$$\Delta W = \Delta W_{DC}\, T \cos \varphi$$

where φ is the synchronous phase measured from the crest.

Since the suitable function (sine or cosine) has been determined for the fields, the NTTF profile can be determined by calculating the NTTF for each β.

$$NTTF = \frac{\Delta W(\beta)}{\Delta W_{\beta_{opt}}}$$

The β that produces the maximum energy exchange is the β_{OPT} and the corresponding ΔW for that β is $\Delta W_{\beta_{opt}}$. The NTTF profile plots for the respective fields are shown below.



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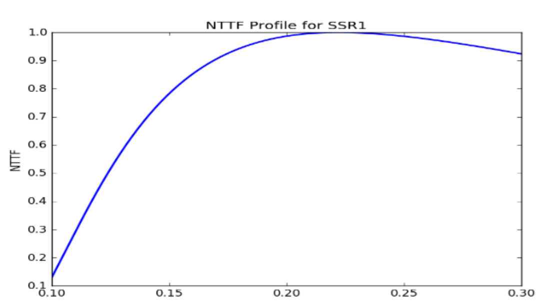

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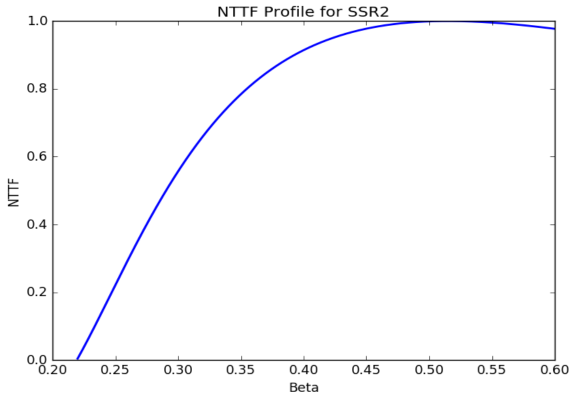

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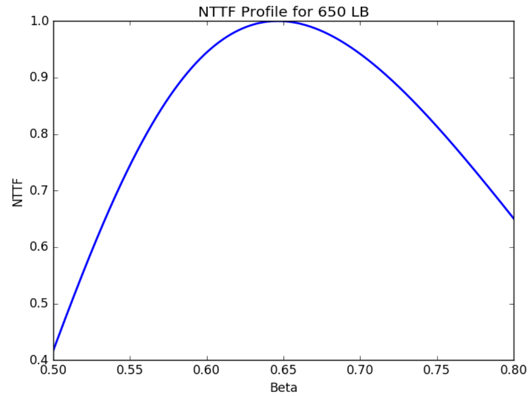

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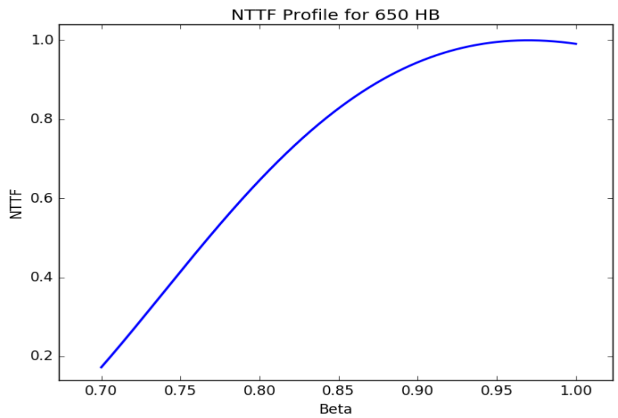


(e)

Figure 6.2: *The NTTF profile for the (a) 162.5 MHz field (2.1 -10 MeV) (b) 325 MHz field (10 - 30 MeV) (c) 325 MHz field (30 - 180 MeV) (d) 650 MHz field (180 - 490 MeV (e) 650 MHz field (490 MeV – 1 GeV)*

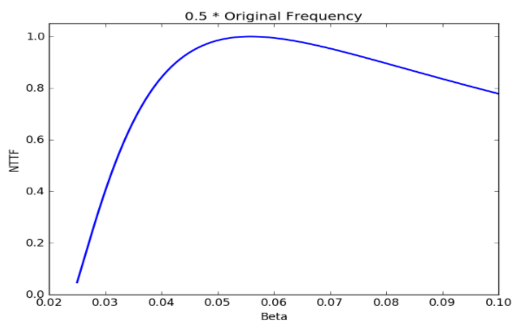
Cavity Type	β_{OPT}
HWR (2.1 – 10 MeV)	0.1118
SSR1 (10 - 30 MeV)	0.2222
SSR2 (30 – 180 MeV)	0.5148
650 LB (180 – 490 MeV)	0.6462
650 HB (490 MeV – 1 GeV)	0.9700

Table 3: The β_{OPT} for each Cavity estimated from the NTTF curves

A very important point to note is that for a cavity, the β is directly proportional to the frequency f , given that other factors are kept constant. Using the different fields, the result is shown on the table below

Cavity Type	β_{OPT}		
	$\frac{f}{2}$	f	$2 * f$
162.5 MHz (2.1 – 10 MeV)	0. 0559	0.1118	0.2236
325 MHz (10 - 30 MeV)	0.1111	0.2222	0.4444
325 MHz (30 – 180 MeV)	0.2574	0.5148	
650 MHz (490 MeV – 1 GeV)	0.4850	0.9700	

Table 4: *The relationship between* β_{OPT} *and f*



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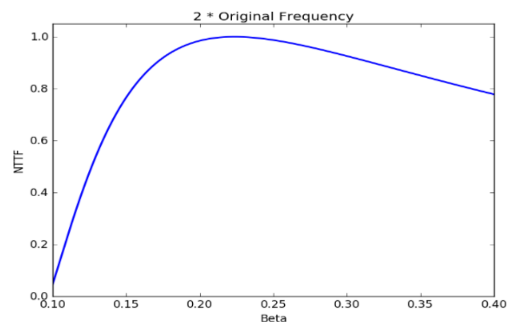

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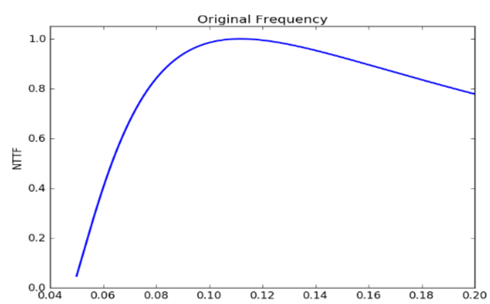


(c)

Figure 7: *The relationship between* β_{OPT} *and f illustrated with 162.5 MHz Cavity*

The graphs are identical, the only difference being that the range of values for the β is exactly half and twice the original range for the '0.5 * frequency' and the '2 * frequency' respectively.

3.4 The Number of Cavities Required for Acceleration

3.4.1 Maximum Energy Exchange: Peak Fields

Relationship between the Kinetic energy of a particle and its β

$$K.E = (\gamma - 1)m_0c^2, \qquad \gamma = \frac{K.E + m_0c^2}{m_0c^2}$$

$$where\ rest\ mass\ energy = m_0c^2 \quad and \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

The rest mass energy for proton = 938.2720813 MeV

$$\gamma = \frac{K.E\ (in\ MeV) + 938.2720813}{938.2720813}$$

Given the NTTF profile for a cavity and the ΔW_{max}, the equations above can be used to determine the number of cavities required to accelerate the protons from one K.E to another. The maximum energy exchange (ΔW_{max}) for a cavity depends on the maximum surface fields that the cavity can withstand (i.e. H_{pk}, the peak surface magnetic field and E_{pk}, the peak surface electric field).

The maximum value for the magnetic field at a given temperature is known as the critical magnetic field and is given the symbol H_c. For all superconductors, there exist a region of temperatures and magnetic fields within which the material is superconducting. Outside this region the material is normal [2]. The H_{pk} must not exceed the RF critical magnetic field (H_c^{rf}) in order to prevent the superconductor from quenching.

When an external magnetic field is applied to a Type I superconductor, the induced magnetic field exactly cancels the external field until there is an abrupt change from the superconducting state to the normal state. However, Type II superconductors like Niobium (which is the superconducting material used for the SRF Cavities) have two critical magnetic field levels, H_{c1} and H_{c2}. Below H_{c1}, Type II superconductors in an increasing magnetic field exclude all magnetic field lines. At field strengths between H_{c1} and H_{c2}, the magnetic field inside the material is not zero. When this occurs the material is said to be in the mixed state, with some of the material in the normal state and part still superconducting. Above H_{c2}, the material goes into the normal state where the flux is able to penetrate the material [2].

The E_{pk} is limited by field emission, which is the emission of electrons from the regions of high electric field on the cavity surface. These electrons absorb RF power thereby reducing the amount of energy the beam gets as it passes through the cavity and consequently, the Q factor. To maximize the accelerating field, it is therefore important to minimize the ratio of the peak fields to the accelerating fields [3].

3.4.2 Accelerate a Proton from 2.1 to 10 MeV with 162.5 MHz HWR Field

The NTTF profile is multiplied by the ΔW_{max} to give the appropriate Energy Exchange profile for a given cavity type. In this case, the ΔW_{max} is 1.5 MeV (see Table 5). So for any given β, the amount energy that will be exchanged when the particle passes through one cavity is given by

$$\Delta W = \Delta W_{max} * NTTF(\beta)$$

Cavity Type	$\Delta W_{max}(MeV)$
HWR (2.1 – 10 MeV)	1.5
SSR1 (10 - 30 MeV)	2.3
SSR2 (30 – 180 MeV)	5
650 LB (180 – 490 MeV)	11.5
650 HB (490 MeV – 1 GeV)	17.4

Table 5*: The Cavity Types and their corresponding* ΔW_{max}.

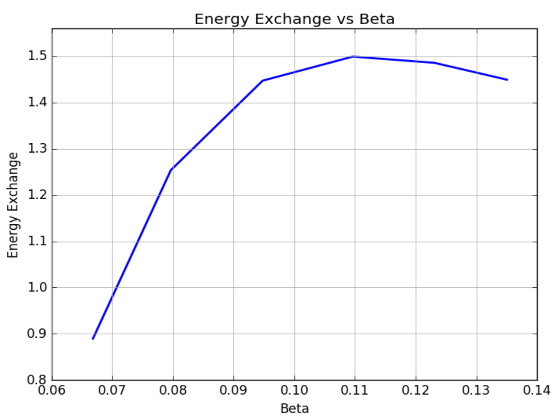
K.E can be converted to β (for use in the equation above) and vice versa based on the relationship between the K.E of a particle and its β that was described earlier. Every time the particle passes through a cavity, the particle gains energy equal to $\Delta W,$ which depends on the β of the particle. After the particle goes through a cavity, the β of the particle changes due to the increase in the K.E. The new value of the K.E will then determine the β of the particle in the next cavity, which will consequently determine the ΔW in the next cavity. This process is repeated until the particle reaches or exceed the desired final K.E (10 MeV in this case).

After running the simulation by performing the calculations above for the HWR, the total number of cavities required to increase the K.E of protons from 2.1 MeV to 10 MeV was determined.

Result: Number of cavities needed = 6

Some plots help to describe the simulation.

- The Energy exchange (ΔW) vs Beta (β): this should be similar to the NTTF curve because the ΔW, which depends on the β in the cavity, is determined from the NTTF profile which is a function of β.
- The Energy Exchange (ΔW) vs # of Cavities: this shows the amount of energy gained by the particles per cavity. It shows this relationship across all the cavities used.
- Total K.E. vs Beta (β): since there is an initial K.E and a desired final K.E for each cavity field, it is good to be able to visualize how the K.E increases from the initial value to the final value with respect to the β of the particles. This plot shows that relationship.
- Total K.E. vs # of Cavities: this shows the same relationship described above in the case of the K.E vs β, but with respect to the number of cavities. It shows the progression of the K.E from cavity to cavity.
- Beta (β) vs # of Cavities: this plot shows the how the β of the particles increases from cavity to cavity. It helps to describe the relationship between particle β and the number of cavities used.



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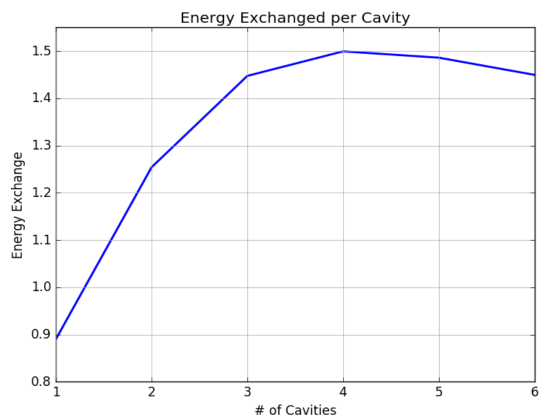

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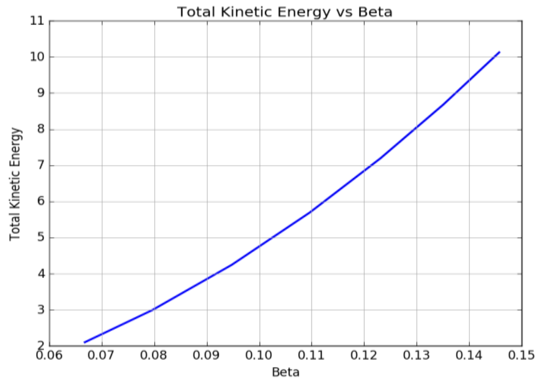

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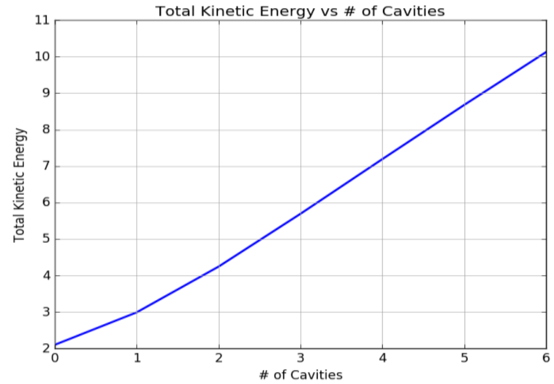

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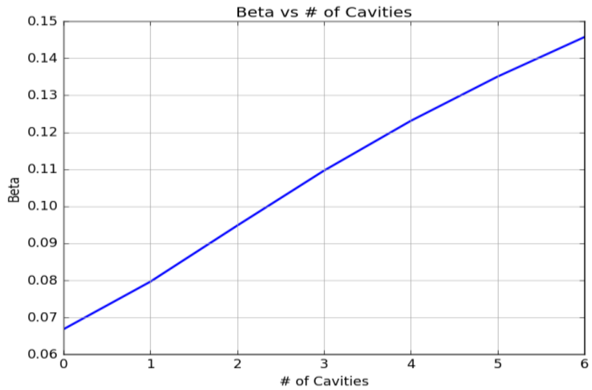


(e)

Figure 8: *(a) The Energy exchange (ΔW) vs Beta (β) (b) The Energy exchange (ΔW) vs # of Cavities (c) Total K.E. vs Beta (β) (d) Total K.E. vs Beta # of Cavities (e) Beta (β) vs # of Cavities*

Cavity Type	# of Cavities
HWR (2.1 – 10 MeV)	6
SSR1 (10 - 30 MeV)	10
SSR2 (30 – 180 MeV)	40
650 LB (180 – 490 MeV)	30
650 HB (490 MeV – 1 GeV)	43

Table 6: *The Cavity Types and their corresponding # of Cavities required for Acceleration*

The same procedure used to determine the number of cavities required for acceleration for the HWR was followed to determine the number of cavities for the other cavity types. Table 6 gives the result of the simulation. The plots for the other cavity types are available in the appendix.

4. Conclusion

Given an accelerating RF cavity longitudinal electric field, maximum energy exchange (ΔW_{max}) , the initial K.E and the desired final K.E of a proton (or particle), the number of cavities required to accelerate the particles within the given energy range can be determined by using the simplified model of linac described in this paper. The simulation involves a series of steps, the first of which is to determine the phase that produces the maximum acceleration of the particle beam. Simulating the Energy Exchange profile, which is later rescaled to the NTTF curve. The NTTF curve and the (ΔW_{max}) can then be used to determine the number of cavities required for acceleration by using the relationship between the K.E of the particles and its β.

The simulation described in this report was performed for five different types of cavity, the result of the simulation is presented on Table 6.

5. References

[1] The Full Wiki. *Superconducting Radio Frequency*. Web. Aug 2016.

[2] Oak Ridge National Laboratory. *Fundamentals of Superconductors.* Apr. 1, 1996. Web. Aug 2016.

[3] Padamsee, Hasan, Jens Knobloch, and Tom Hays. *RF Superconductivity for Accelerators.* Morlenbach, Germany: WILEY-VCH Verlag GmbH & Co. KGaA, 2009. p(43) Print.

[4] U.S. Particle Accelerator School. *Lecture 4: RF Acceleration in RF Acceleration in linacs Part 1*. Web. Aug 2016.

6. Appendix

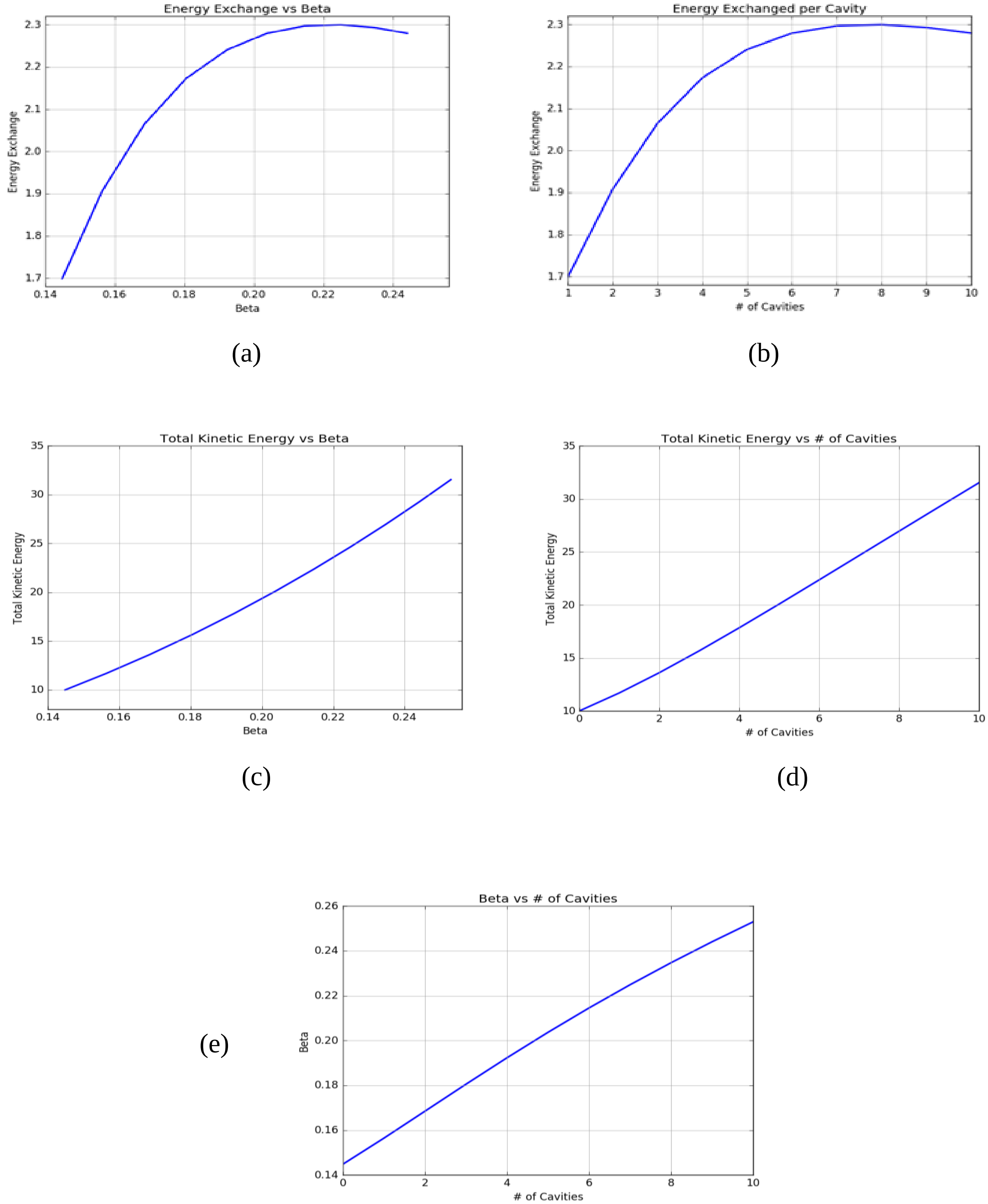
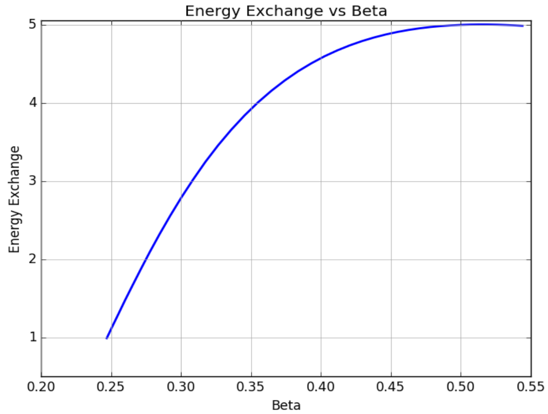


Figure 9: *SSR1 (10 -30 MeV) (a) The Energy exchange (ΔW) vs Beta (β) (b) The Energy exchange (ΔW) vs # of Cavities (c) Total K.E. vs Beta (β) (d) Total K.E. vs Beta # of Cavities (e) Beta (β) vs # of Cavities*



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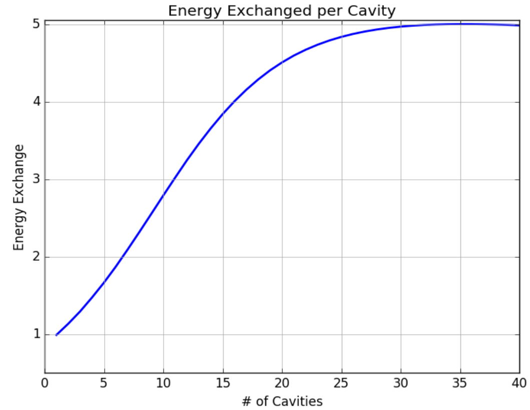

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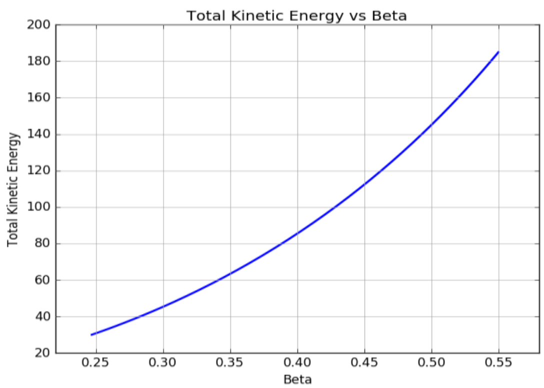

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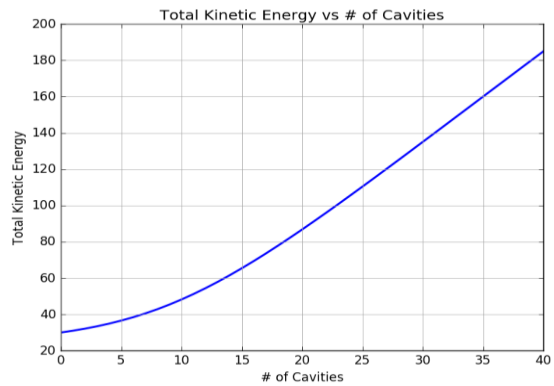

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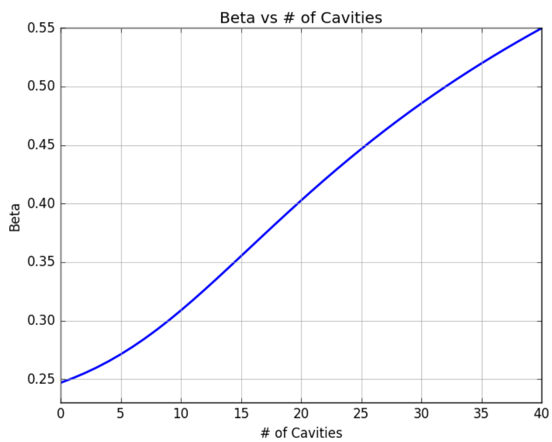
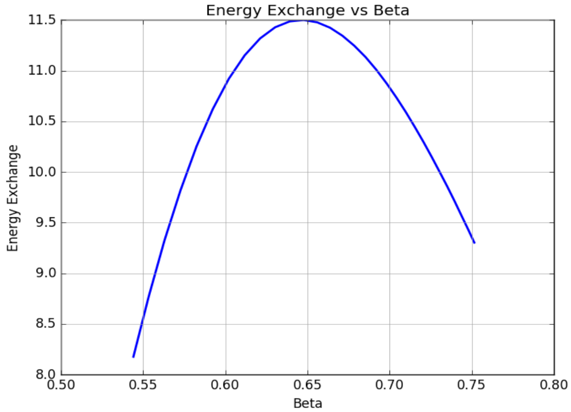


Figure 10: *SSR2 (30 -180 MeV) (a) The Energy exchange (ΔW) vs Beta (β) (b) The Energy exchange (ΔW) vs # of Cavities (c) Total K.E. vs Beta (β) (d) Total K.E. vs Beta # of Cavities (e) Beta (β) vs # of Cavities*



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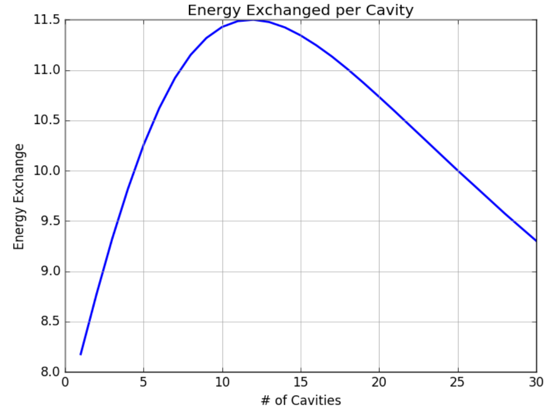

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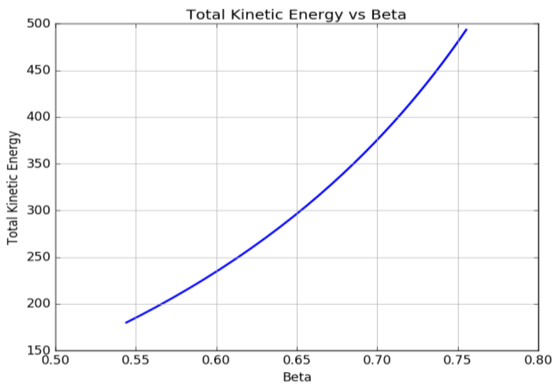

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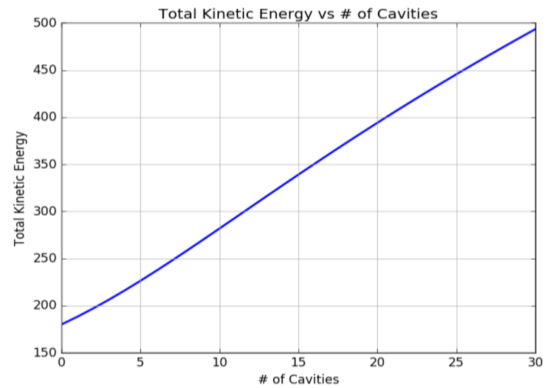

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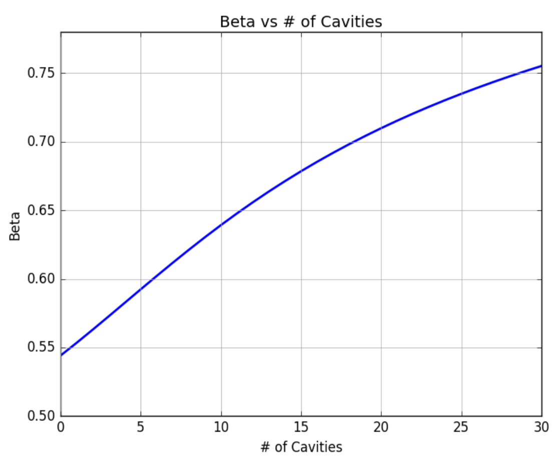
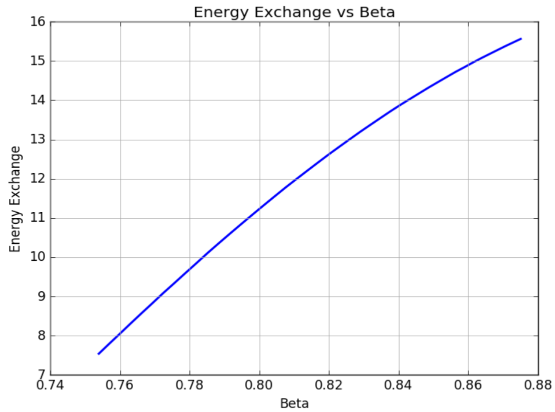


Figure 11: *650 LB (180 -490 MeV) (a) The Energy exchange (ΔW) vs Beta (β) (b) The Energy exchange (ΔW) vs # of Cavities (c) Total K.E. vs Beta (β) (d) Total K.E. vs Beta # of Cavities (e) Beta (β) vs # of Cavities*



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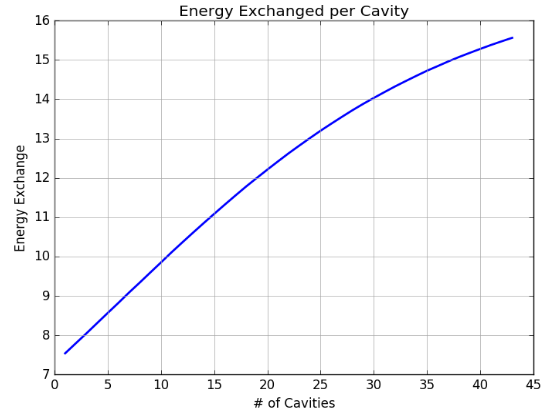

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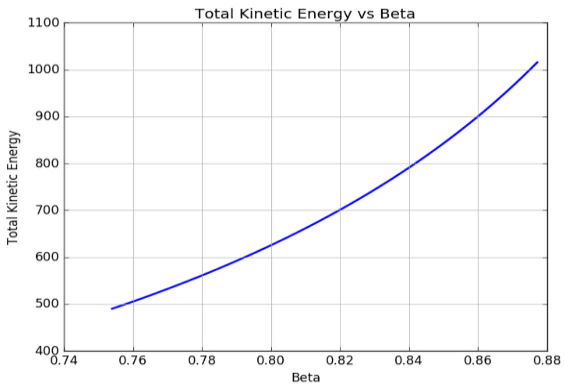

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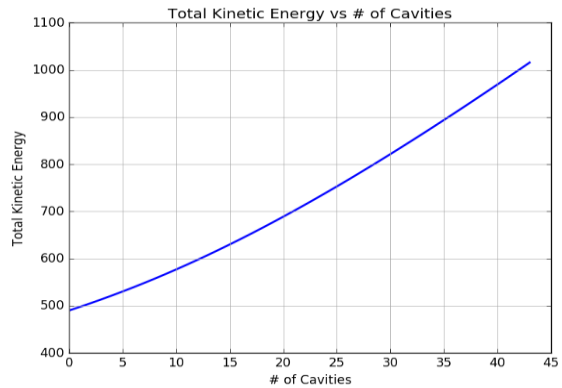

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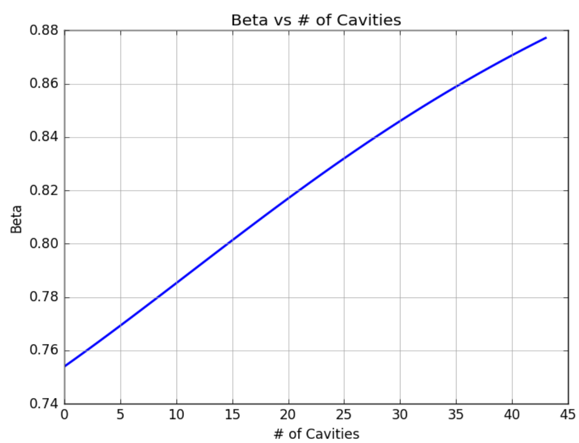

Figure 12: *650 HB (490 MeV – 1 GeV) (a) The Energy exchange (ΔW) vs Beta (β) (b) The Energy exchange (ΔW) vs # of Cavities (c) Total K.E. vs Beta (β) (d) Total K.E. vs Beta # of Cavities (e) Beta (β) vs # of Cavities*