

Diffusion measurement with transverse beam echoes

Yuan Shen Li, Carleton College – Lee Teng Fellowship

Tanaji Sen, Fermilab

Introduction

- Measuring and managing diffusion is crucial in modern “intensity frontier” machines, where nonlinear phenomena, e.g., intrabeam scattering and space charge effects, can significantly increase emittance over time.
- Traditional methods to measure diffusion, e.g., beam scraping, take up to hours to complete. The transverse echo technique will require minutes or less.
- The echo displays high amplitude sensitivity to small phase space perturbations, making it an ideal tool to probe weak diffusion.
- Simultaneously, we need amplitude-boosting techniques to counter strong diffusion (e.g., space charge effects), so that the echo signal remains measurable.
- In this study, we develop theory and simulation to:
 - Explore the behavior of transverse echoes under diffusion.
 - Investigate pulsed quadrupoles as a method to boost echo amplitude.
 - Provide recommendations for the planned beam echo measurement system in the future IOTA storage ring at Fermilab.

Echo: Theory and Simulation

Theory

- The transverse echo is a recoherence of the beam distribution, following phase decoherence due to nonlinear ring elements (e.g., octupoles).
- It shows up on the BPM as an oscillation of the beam centroid, some time after an initial disturbance (e.g., dipole kick).
- Typical echo sequence:
 - At $t = 0$, apply one-turn dipole kick θ .
 - At $t = \tau$, apply one-turn quadrupole kick q .
 - Near time 2τ , the echo signal appears on the BPM.
- The amplitude of the echo is dependent on ring parameters. It is also extremely sensitive to diffusion. (Refer to equations above.)
- Key assumptions:
 - Both dipole and quad kicks are weak (compared to beam spread).
 - The timing of quad kick τ is much greater than decoherence time.

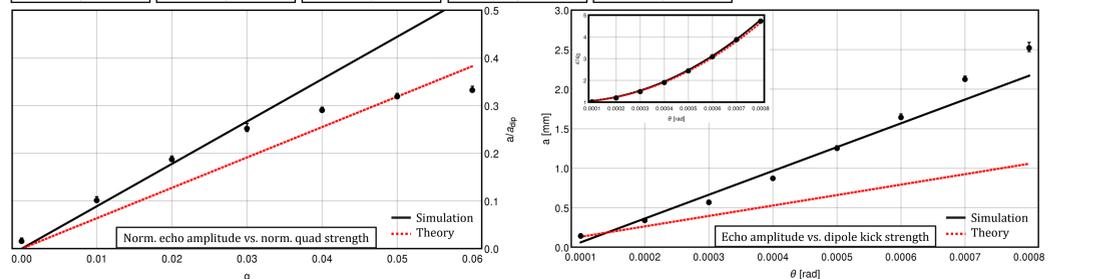
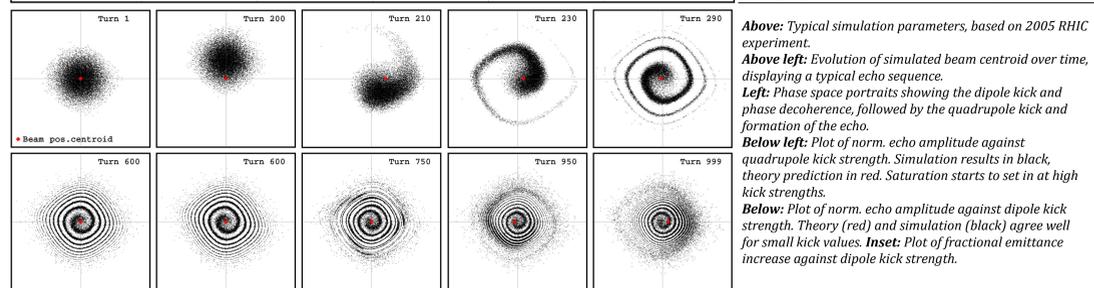
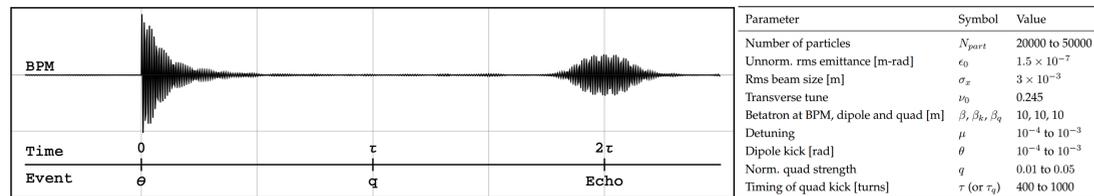
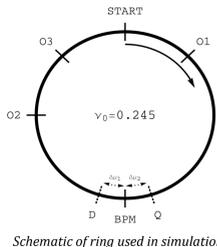
$$a_0 = \theta q \sqrt{\beta_k \omega' J_0 \tau}$$

$$a_{\text{diff}} = \frac{\theta q \sqrt{\beta_k \omega' J_0 \tau q}}{\alpha_1^3}$$

where $\alpha_1 = 1 + \frac{2}{3} \left(\frac{D_1 \tau q}{J_0^2} \right) (\omega' J_0 \tau)^2$

Simulation

- Simulation written in C, with analysis performed in *Mathematica*.
- Machine parameters based on 2005 RHIC experiment.
- Simulation options include adjustable ring elements, variable starting distribution, variable diffusion model, pulsed quadrupoles and injection oscillation.
- Simulation results agree well with theory.

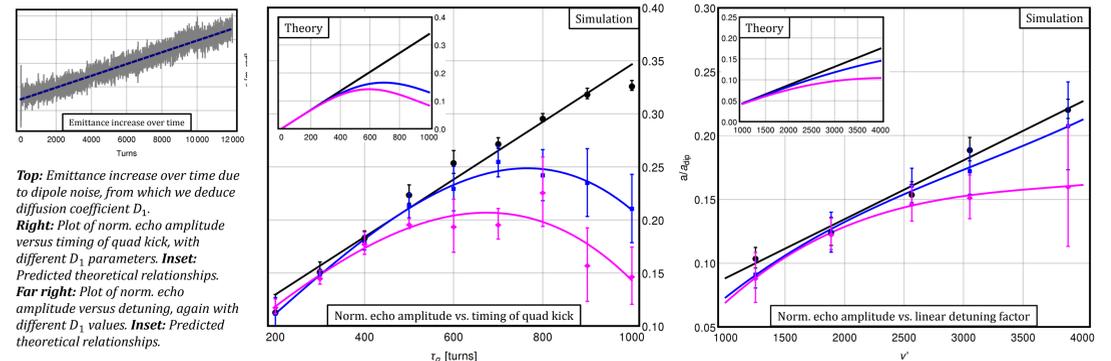


Diffusion

- Linear diffusion model simulated by dipole noise.
- Echo amplitude becomes attenuated with diffusion.
- We directly measure diffusion coefficient by tracking emittance increase over a large number of turns. Results agree excellently with theory.
- Simulation results also demonstrate predicted relationship between echo amplitude and relevant parameters (below).

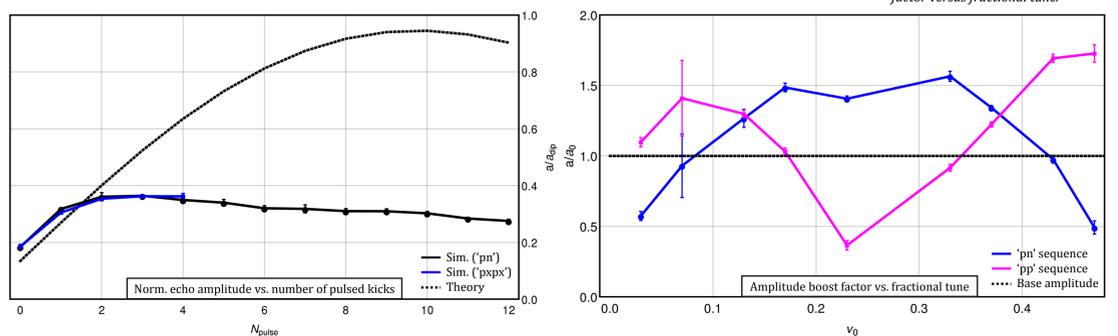
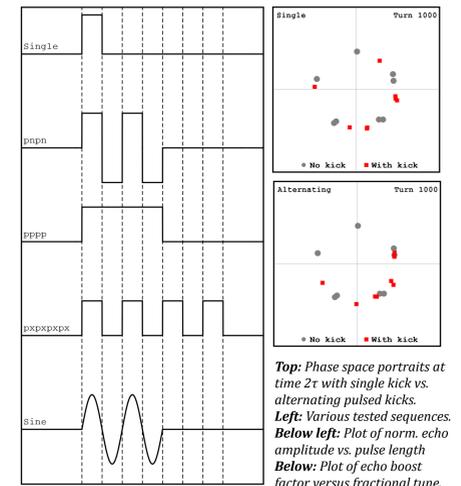
$$\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial J} \left(D(J) \frac{\partial \psi}{\partial J} \right), \quad \text{where } D(J) = D_0 + D_1 \left(\frac{J}{J_0} \right)$$

$$D_1 = \pi \epsilon_0 \frac{d\epsilon}{dt} \quad \tau_{\text{max}} = \left(\frac{16}{3} \omega'^2 D_1 \right)^{-1/3}$$



Pulsed quadrupoles

- Based on gradient echoes in NMR.
- A single quad kick introduces a small, position-dependent ΔJ to the particle distribution. With linear detuning, this leads to particles “clumping” together in phase space at time 2τ .
- Pulsed kicks apply a sequence of small ΔJ ’s that amplify each other, resulting in a tighter “clump” in phase space.
- Optimal sequence highly dependent on fractional tune. We investigated several possible sequences.
- Maximum echo amplification close to 100% (up to saturation point).



Conclusions and Further Work

- Key findings: Consistent measurement of diffusion coefficient based on τ_{max} ; echo amplitude boost by up to 100% using pulsed quads; optimal sequence depends on fractional tune; pulsed sequence of single polarity can be just as effective.
- Some further questions:
 - What is the optimum pulse sequence for a given fractional tune?
 - Echo amplitude saturation observed empirically at $A \approx 0.4$. How do we explain it? Is it possible to surpass this limit?
 - How will echo dynamics change in 2D? Any coupling effects?

Acknowledgements

This research project was made possible by the Lee Teng Fellowship in Accelerator Science and Engineering hosted by Fermilab. Additionally, the author would like to thank his research mentor, Tanaji Sen; internship director, Eric Prebys; and the knowledgeable instructors at USPAS for their invaluable guidance and support.