Extraction of the Muon Beam Frequency Distribution via the Fourier Analysis of the Fast Rotation Signal in the Muon g-2 Experiment

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Background

In the presence of an external magnetic field, fermions acquire a magnetic moment

$$\vec{\mu} = g\frac{Q}{2m}\vec{s}$$

where g is the gyromagnetic ratio, Q the electric charge, and $\vec{s}$ is the fermion's spin vector. The anomalous part a of the magnetic moment is defined via the deviation of the gyromagnetic ratio: $g = 2(1+a)$.

The Fermilab E989 Muon g-2 experiment aims to measure a_μ to a precision of 0.14 ppm[1]. The experiment stores muons inside a weak focusing ring, where a magnetic field and electrostatic quadrupoles (ESQs) provide inward radial and vertical focusing respectively. The magnetic field intensity corresponds to storing muons on the design radius (7.112 m) with a design momentum of 3.094 GeV/c. The measurement of a_μ depends on the muon intrinsic spin precession frequency ω_a, which is sensitive to the above fields. Due to a beam momentum spread of 0.1%, ω_a must be adjusted for the effect of a radial electric field. The correction relies on the equilibrium muon revolution frequency distribution, extracted via a modified Fourier analysis of the so-called fast rotation signal (FRS). The method was developed for E821 at BNL[2], but we independently re-derive several key results, as well as explore the numerical extension of the method, and compare simulation results to data from the recent commissioning run.

Theoretic Results

Background

- $\vec{\omega}_a = -\frac{Qe}{m}\left[a_\mu\vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$ radial E-field contribution
- The FRS describes the beam intensity as seen by a detector at a fixed location in the ring. For a longitudinally point-like beam with momentum spread $\rho(\Delta)$, the FRS is given by

$$S_0(t) = \sum_{n=0}^{\infty}\frac{\rho\left(\frac{t}{nT+t_0}-1\right)}{nT+t_0}$$

t_0 is the time when the center of mass of the beam first passes the detector. The muon revolution frequency distribution is given by the Fourier transform

$$\widehat{S_0}(\omega) = \sqrt{\frac{2}{\pi}}\int_{t_0}^{\infty} S_0(t)\cos\omega(t-t_0)\,dt$$

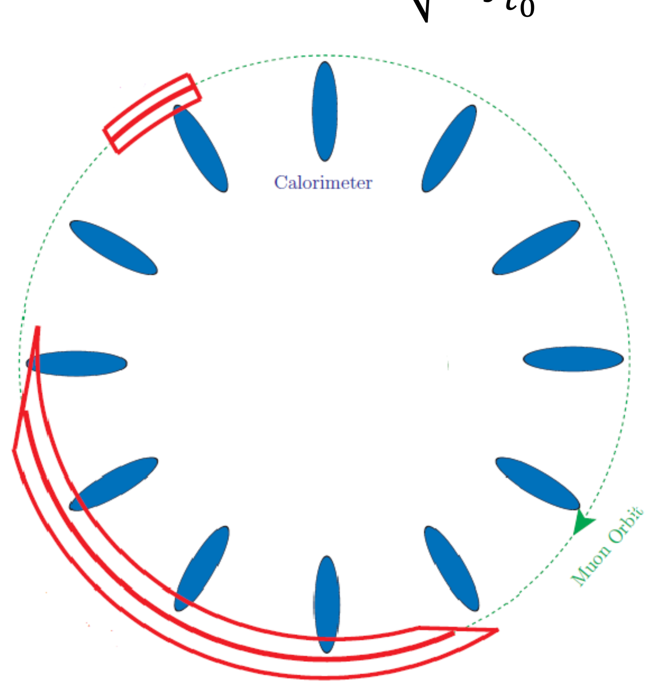

Figure 1. Depiction of a muon beam debunching with time due to its momentum spread. The short bunch is for $t \approx t_0$, and the longer bunch for $t \gg t_0$.

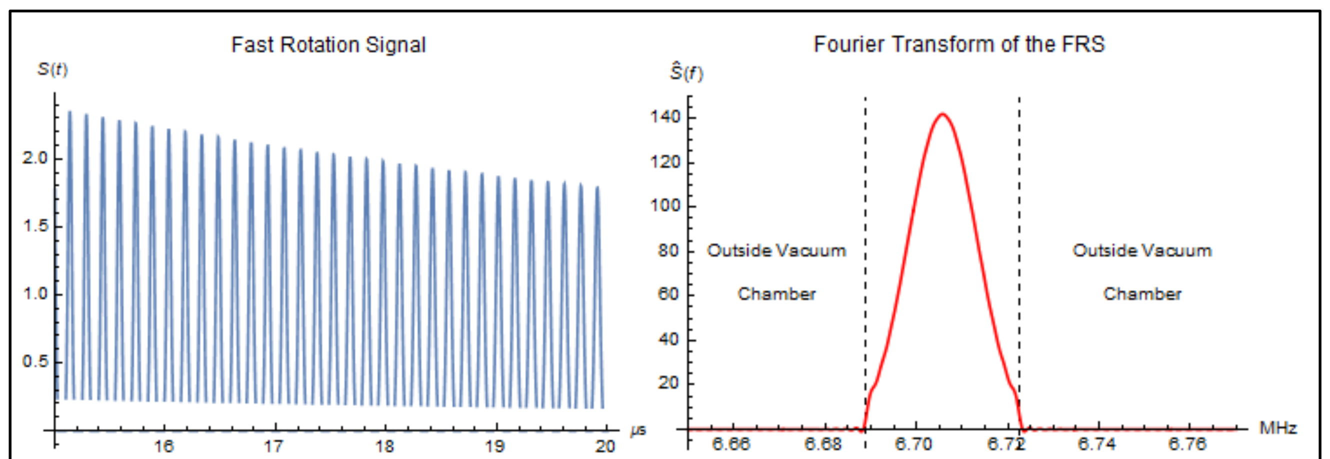

Figure 2. Left, FRS for a point-like beam (longitudinally and transversally) with 0.112% momentum. Right, the corresponding Fourier transform.

Longitudinal Bunch Profiles

- Given a longitudinal profile $\xi(t)$, the FRS is expressed from the convolution: $S(t) = \sqrt{2\pi}\xi(t) * S_0(t)$.
- By the convolution theorem $\hat{S}(\omega) = \sqrt{2\pi}\hat{\xi}(\omega)\widehat{S_0}(\omega)$. A non-even $\xi(t)$ yields an imaginary $\hat{S}(\omega)$ and thus an imaginary correction to ω_a.
- Redefine $t_0 \to t_0 + x_0$ where x_0 is the root of

$$f(x) = \int_{\xi_- - x}^{\xi_+ - x}\xi(t+x)\sin(\omega t)\,dt$$

and (ξ_-, ξ_+) the domain of ξ.
- Use of $t_0' = t_0 + x_0$ and $\xi'(t) = \xi(t+x_0)$ for the Fourier analysis, yields the correct $\hat{S}(\omega)$.

Corrections to the Fourier Transform

- Beam detection at time $t_s > t_0$ yields an incomplete frequency spectrum $\widehat{S_1}(\omega)$, requiring an approximation for

$$\Delta(\omega) = \sqrt{\frac{2}{\pi}}\int_{t_0}^{t_s} S(t)\cos\omega(t-t_0)\,dt$$

- If $t_s - t_0$ is sufficiently small, the Fourier uncertainty principle ($\Delta\omega\Delta t \sim 2\pi$) implies that $\widehat{S_1}(\omega)$ will have a spread comparable to $\hat{S}(\omega)$, retaining the same information as the latter. Using the inverse Fourier transform and $\widehat{S_1}(\omega)$, $\Delta(\omega)$ may be approximated as:

$$\Delta(\omega) \approx \frac{1}{\pi}\int_{\omega_-}^{\omega_+}\widehat{S_1}(\omega')\frac{\sin[(\omega-\omega')(t_s-t_0)]}{\omega-\omega'}\,d\omega'$$

where (ω_-, ω_+) corresponds to the range of revolution frequencies allowed in the vacuum chamber.

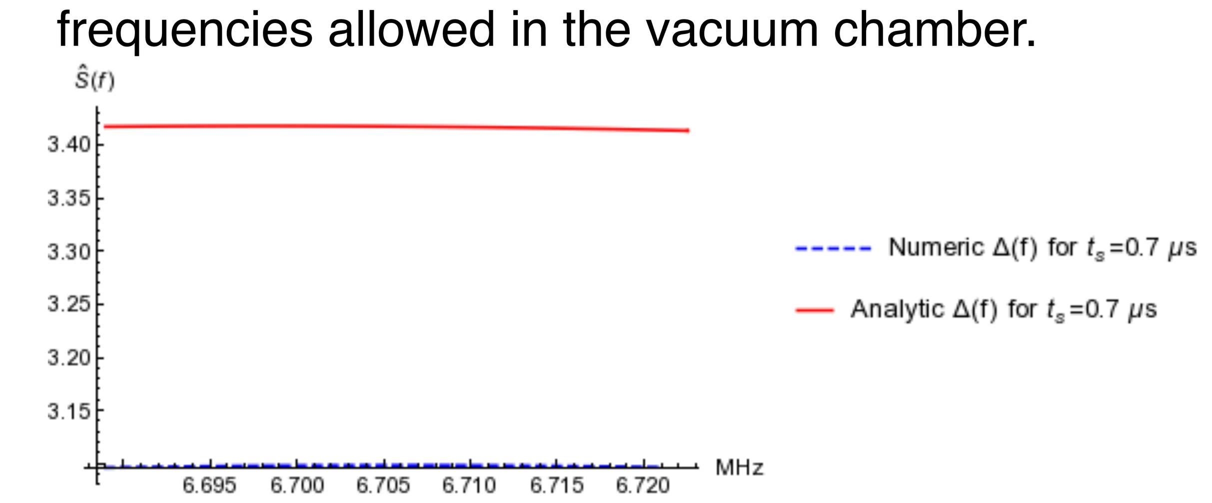

Figure 3. Comparison of the exact and approximated $\Delta(\omega)$ for the beam described in Figure 2. Their spreads are 0.0100987 and 0.0100985 MHz respectively.

The Numerical Extension

- Discrete Fourier transforms (DFTs) used on discrete data
- FRS properties require the first entry of the DFT to correspond to a bin with t_0 as its midpoint.
- The data analysis framework ROOT (version 6.06) performs the DFT using the built-in FFTW C library.

Simulation vs. Real Data

- Simulation conducted using the BMAD subroutine library; actual data acquired from scintillating-fiber beam monitor 2 in commissioning run #1835
- Predominantly protons in the fill

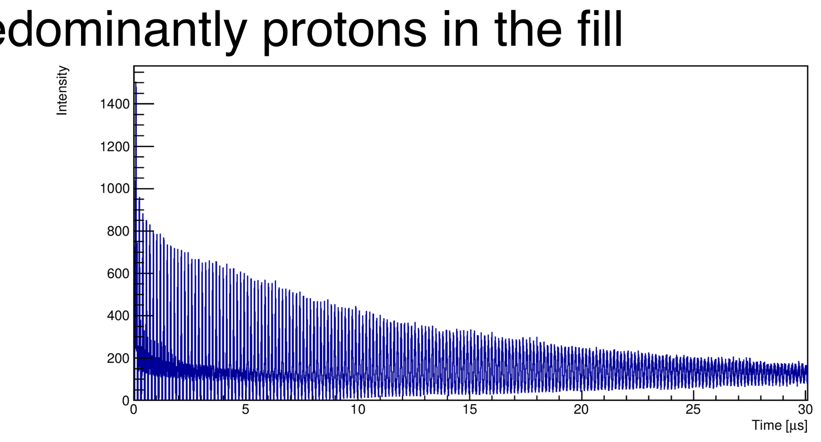

Figure 4. The FRS from BMAD.

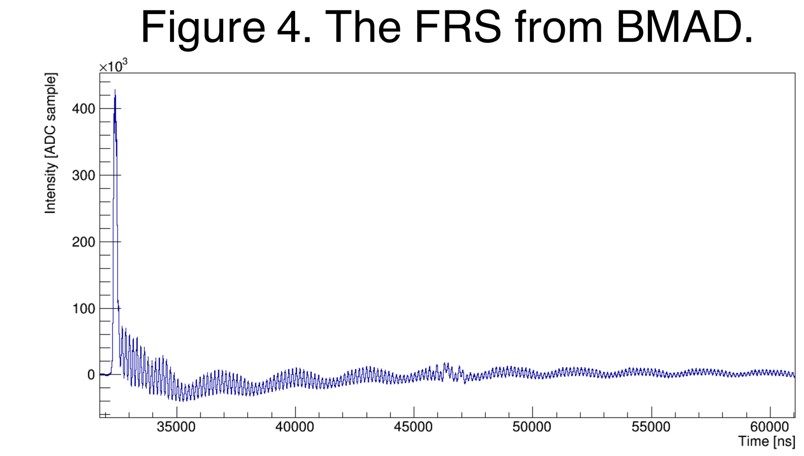

Figure 5. The FRS from real data. The beam was injected at ~ 32 μs. The noise at 47 μs is due to the ESQs gaining higher voltage. The baseline undershoot is a detector effect.

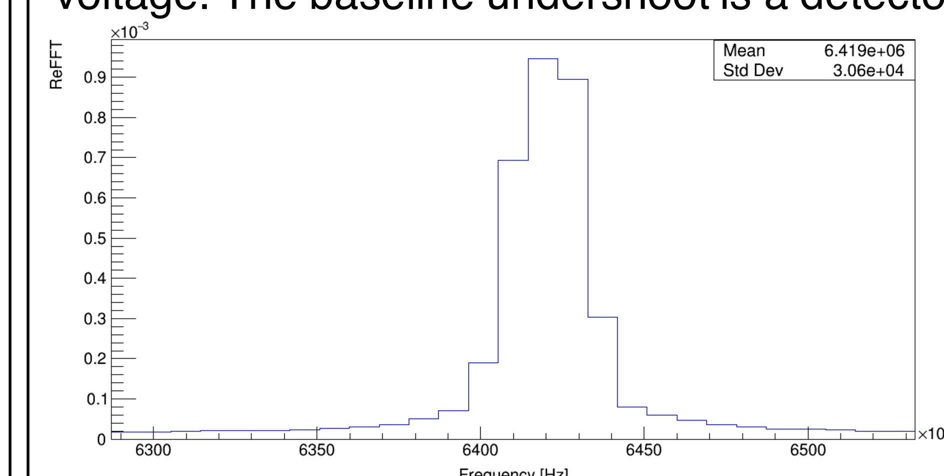

Figure 6. DFT of simulation.

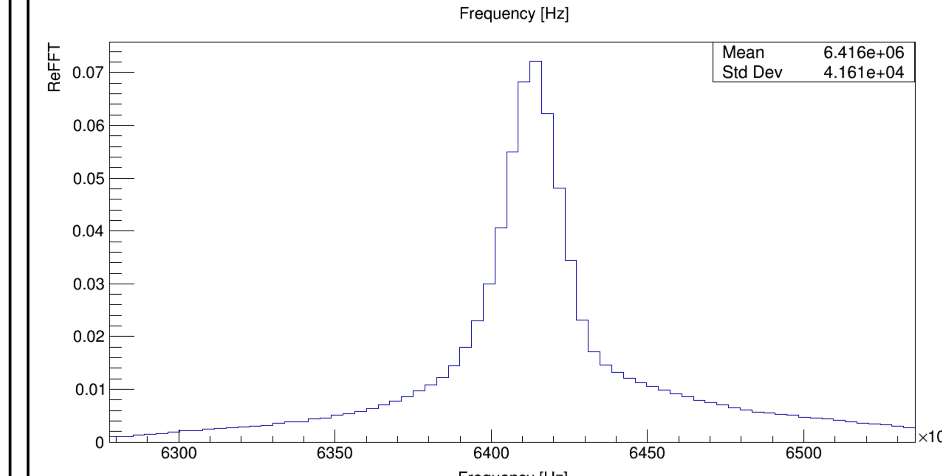

Figure 7. DFT of real data.

- The peaks obtained from simulation and real data feature a mismatch of ~ 757 kHz.

Future Work

- Studies on how muon losses and coherent betatron oscillations impact the frequency distribution.
- Most FRS data will come from calorimeter stations; must explore the analysis using calorimeter stations.

References

[1] J. Grange et al. *Muon (g−2) technical design report* (2015), arXiv:1501.06858 [physics.ins-det].

[2] Yuri Orlov, Cenap S. Ozben, and Yannis K. Semertzidis. "Muon revolution frequency distribution from a partial-time Fourier transform of the g-2 signal in the muon g-2 experiment," *Nuclear Instruments and Methods in Physics Research* A482 (2002) 767775.

Acknowledgments

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