Calibration of single-photon detectors using correlated beams from spontaneous parametric
down-conversion

Justin Ripley
Office of Science, Science Undergraduate Laboratory Internship (SULI) Program

Columbia University, New York

Fermi National Accelerator Laboratory
Batavia, Illinois

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in the Particle Physics Division at Fermi National Accelerator Laboratory.

Participant: ________________________________
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Research Advisor: ________________________________
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A metrology lab that utilizes correlated photons from spontaneous parametric down-conversion to calibrate photodetectors was put together at Fermilab. The lab was made specifically to calibrate silicon photomultipliers, a new kind of photodetector device with a broad range of applications in modern particle physics experiments. In addition to the alignment of the optical equipment in the lab, the necessary software was written and tested and an experimental protocol was devised in order to efficiently calibrate silicon photomultipliers purchased by experimenters at Fermilab.

I. INTRODUCTION

Spontaneous parametric down-conversion (SPDC) allows for the absolute calibration of single-photon detectors. SPDC involves the near simultaneous creation of pairs of photons in a nonlinear crystal. In the process, a pump photon at frequency $\omega_p$ and momentum $k_p$ splits into two down-converted photons (the idler and signal) at frequencies $\omega_i$ and $\omega_s$ and momenta $k_i$ and $k_s$, respectively. Momentum and energy conservation dictate that the three beams must satisfy the four conditions: $\omega_p = \omega_i + \omega_s$ and $k_p = k_i + k_s$. The wavelengths of both down-converted photons are then correlated. Due to the nonlinear crystal susceptibility $\chi^{(2)}$ of the crystal, photons that are correlated at specific wavelengths emerge at specific angles with respect to the crystal’s optical axis. By rotating the optical axis of the crystal with respect to the pump beam, correlated down-converted photon pairs at a given wavelength can be directed to two separate photodetectors (the trigger and conjugate). Given that the trigger detector measures one photon (the idler), there is a guarantee that the other photon (the signal) is present at the conjugate detector [1]. For $N$ down-converted photon pairs produced at a specific wavelength and with a quantum efficiency of $\eta_t$ and $\eta_c$ for the trigger and conjugate detectors respectively, the number
of photons measured by the trigger will be \( N_t = N \eta_t \) and the number of photons measured by the conjugate detector \( N_c \) given a detection by the trigger will be \( N_c = N \eta_t \eta_c \). The efficiency of the conjugate detector is then equal to:

\[
\eta_c = \frac{N_c}{N_t}
\]

This technique was first proposed by Klyshko [2] and known as the Klyshko method. The first photodetector calibrations using this method, performed by Rarity et al. [3] and Kwiat [4], demonstrated the usefulness of the Klyshko method in specifically calibrating single photon detectors. The method agrees well with non-SPDC calibration techniques for photomultiplier tubes that rely on outside calibration standards [5], [6]. In an actual experiment other factors such as dark counts, accidental coincidences, and optical path losses to the detectors must be taken into account to find the true efficiency of the conjugate detector.

Silicon photomultipliers (SiPM) are a new photodetection technology that allows for single photon resolution and counting [7]. Despite their great utility, to our knowledge there has been no calibration of SiPMs using spontaneous parametric down-conversion. A photodetector characterization room that uses SPDC was set up in order to test the photodetection efficiency of SiPMs. In addition to the calibration of these SiPMs, the characterization room can serve as a photodetector metrology lab for Fermilab that will allow for the independent verification of the efficiencies of photodetectors bought by Fermilab from industry. All necessary steps except the accurate fine alignment of the SiPM mounts were completed, and several preliminary tests of the detectors used for alignment purposes were completed.

II. MATERIALS AND METHODS

A. Modified calibration formula
The Klyshko method was used in calibrating the silicon photomultipliers via SPDC. Any time a SiPM registered a voltage response that exceeded the set threshold will be called a count. In an ideal experiment with no dark pulses and stray photons, the efficiency of the conjugate detector would be equation (1). In a realistic experiment, equation (1) has to be modified to take into account other relevant factors that could cause the trigger and conjugate detectors to register a count even if no actual down-converted photon hit the detectors. Besides down-converted photon pairs in the room, background counts $B'_t$ will be registered by the trigger detector in a given run. Thus the true number of down-converted photons reaching the trigger detector during each calibration run is $N_t - B'_t$, where $N_t$ is the total number of counts registered by the trigger detector. Note that $B'_t$ includes counts caused by actual background photons in the room during the calibration measurement and random dark pulses caused by thermal excitation of electrons in the SiPM. In addition to random single counts by the trigger detector, background coincident counts $B'_c$ must be taken into account as well. As with the singles background counts $B'_t$, the coincident background counts term $B'_c$ takes into account any background event that was not caused by two down-converted photons that reached both detectors simultaneously. The modified efficiency formula for the efficiency of the conjugate detector is then:

$$\eta_c = \frac{N_c - B'_c}{N_s - B'_t}$$ (2)
B. Experimental setup

A schematic of the experimental setup is shown in Fig. 1; each element in the figure is explained below.

1) The source of the pump beam (1) was a semiconductor CW gallium-arsenide 404-nm laser (Cube 405, made by Coherent). The beam was vertically polarized and operated at 100 MW. A 7-cm outer diameter iris (2) was used to remove unwanted laser fluorescence. Two 3-cm outer diameter irises, (4) and (5), followed which further collimated the beam. The width of the apertures of the irises was modified throughout and the alignment runs to experiment with reducing laser fluorescence. In the initial alignment a double Glan-Taylor polarizer was placed in between components (3) and (4) in order to verify the vertical polarization of the pump beam.

2) A 5x5x5 mm beta-barium borate (BBO) crystal was used for the nonlinear crystal (6). The crystal was cut with its optical axis at 29.5° with respect to the pump beam for type 1
phase matching. After the crystal a mirror deflected the remaining pump beam to a beam dump (7).

3) Two 810-nm filters, (8) and (9), with spectral widths of ±10 nm were placed in front of both the trigger (10) and conjugate (11) SiPMs. The SiPMs were each aligned at 3° from the pump wave vector and placed on translational mounts for fine alignment. The detectors were mounted 1 m away from the BBO crystal, and were separated by a distance of roughly 10 cm.

4) Each detector was connected to a field-programmable gate array (FPGA) SiPM general-purpose readout board [8]. SiPM1 was connected to channel zero of the board, and SiPM2 was connected to channel one. The board was modified for low-bandwidth processing, and the version was test board four (TB_4). The trigger detector, conjugate detector and coincidence counts were collected 18.8 ns bins. A typical calibration run would include the results of 500 bins.

C. General experimental procedure

The two detector mounts were first roughly aligned geometrically. Two single pixel (100 μm) SiPMs were then used for fine alignment of the detector mounting stages; the trigger detector was finely aligned by adjusting the position of the detector on the translational stage with the pump beam on until maximal single count rates were measured. The conjugate detector was then adjusted until maximal coincident count rates were measured. Once the maximum count rate positions were located, the dark count rates for the two alignment SiPMs were measured, along with the count rates for the detector with the laser running through the BBO crystal and with the laser not running through the BBO crystal to determine the number of photons produced detected that were produced by the pump beam fluorescence intrinsic to diode
lasers. Laser fluorescence was reduced through the use of various irises as well as a large box
(not shown in Fig. 1) that covered components (1)–(6.) The box had an opening (~100 cm²) that
allowed the pump and down-converted beams to exit.

D. Experimental determination of variables in modified calibration formula

The background events term $B_t'$ is calculated in a background run by measuring the total
number of events registered by the trigger photon with the pump laser off for a time $\tau_b$, with $B_t$
events registered in that time window. The calibration run with the pump laser on would last for
a time $\tau_{dc}$, and with $\tau_r = \tau_{dc} / \tau_b$, the estimated number of background events during the calibration
run is estimated to be $B_t' = B_t \tau_r$. The background coincidences (accidentals) are determined by
introducing a specified time delay between the trigger and conjugate detectors with the laser on,
and counting the number of coincidences $B_c$ in a time run of length $\tau_c$ during the accidentals run.
Letting $\tau_r' = \tau_{dc} / \tau_c$, the number of coincidence counts expected during the calibration run is $B_c' = B_c \tau_r'$. Generally $\tau_{dc}$, $\tau_c$, and $\tau_r$ will all be roughly equal to each other. Using the above measured quantities the modified efficiency formula for the conjugate detector then becomes:

$$\eta_c = \frac{N_c - B_c \tau_r'}{N_s - B_t \tau_r} \quad (3. a)$$

Due to the filter used in front of the conjugate detector, formula (3.a) must be divided by
the filter efficiency $\rho$. In addition, for calculation of the uncertainty of the above value, the
number of “anticoincidence counts” $S_a = N_s - S_c$ (where $S_c$ is the number of coincidence counts)
must be included in formula (3.a) giving then:

$$\eta_c = \frac{N_c - B_c \tau_r'}{\rho (S_c + S_a - B_t \tau_r)} \quad (3. b)$$

While the background and accidental photon count rates follow a Poisson distribution,
due to the expected large amount of the rates encountered in calibration experiments (typical
rates were over 100,000 counts per second for the alignment SiPMs with the pump laser on with no crystal in place), the relative standard deviation around the mean will be very small. This allows for a simplified multiplication formula for the background rates (see Appendix).

The relative error in the calculated efficiency is determined by assuming that the accidentals count rate $B_c$, the background trigger rate $B_t$, the filter efficiency $\rho$ (with value uncertainty $\Delta \rho$) and the anticoincidence value $S_\alpha$ all followed independent Poisson distributions. The relative error formula is then (see appendix):

$$
(\Delta \eta)^2 = \left(\frac{1}{\rho(S_c + S_\alpha - B_t \tau_r)}\right)^2 (B_c \tau'_r) + \left(\frac{N_c - B_c \tau'_r}{\rho(S_c + S_\alpha - B_t \tau_r)}\right)^2 (B_t \tau_r) 
+ \left(\frac{N_c - B_c \tau'_r}{\rho(S_c + S_\alpha - B_t \tau_r)}\right)^2 (S_\alpha) + \left(\frac{N_c - B_c \tau'_r}{\rho^2 (S_c + S_\alpha - B_t \tau_r)}\right)^2 (\Delta \rho)^2
$$

(4)

The anticoincidence value $S_\alpha$ is the number of times that the trigger detector registered an event, but the conjugate detector failed to register an event.
III. RESULTS

FIG. 2. Latest metrology lab setup.

Figure 2 displays the current metrology lab setup without the laser-fluorescence shielding. Note that the BBO crystal is not in the mounting stage, and there is no electronic shielding over the TB_4 board.

The event count rates for the alignment SiPMs, once they were aligned for maximum single and coincidence count rates, with the pump beam running through the BBO crystal, are displayed in Table 1. The detection count rate data was collected over 500 event runs (500 bins). The error was calculated to one standard deviation assuming the data followed a Poisson distribution.
TABLE I. Detection counts per second with alignment SiPMs (500 bin run)

<table>
<thead>
<tr>
<th></th>
<th>Channel 0</th>
<th>Channel 1</th>
<th>Coincidences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laser off</td>
<td>179 500 (± 423)</td>
<td>173 344 (± 416)</td>
<td>158 859 (± 399)</td>
</tr>
<tr>
<td>Laser on, with BBO</td>
<td>1 613 297 (± 1 270)</td>
<td>2 995 289 (± 1 731)</td>
<td>286 846 (± 536)</td>
</tr>
<tr>
<td>Laser off, without BBO</td>
<td>1 579 296 (± 1 257)</td>
<td>4 180 864 (± 2 045)</td>
<td>609 153 (± 780)</td>
</tr>
</tbody>
</table>

IV. DISCUSSION

A. Noise and background counts

In Table I, it is readily seen that the overall background and coincidence counts increased when the crystal was removed. The BBO crystal used is housed in a large plastic case, which probably blocked out excess laser fluorescence from hitting the alignment detectors. The fact that the counts are so much higher for the detectors without the crystal indicates that the current setup of diaphragms and dark boxes is insufficient in reducing laser fluorescence to acceptable levels. Laser fluorescence cannot be controlled directly and thus it is crucial for that source of background to be eliminated as much as possible.

B. Future work

A new dark box that will enclose the SiPMs and their translation mounts is needed to reduce general background radiation counts on the SiPMs. Background rate tests performed with smaller mounts and a dark box showed that a dark box with two small openings for the down-converted light reduced background count rates by over an order of magnitude. In addition, a dark glass filter (not yet purchased) placed immediately after the pump laser should also significantly decrease fluorescence radiation.
Before accurate efficiency measurements of SiPMs can be made, the fine alignment of the detector mounts must be further verified by shifting the positions of the mounts about the measured local maximum count rate positions. The translations should ideally occur in three orthogonal directions, while the current detectors are mounted on translation stages that can only move along one direction. Three directions of detector mount adjustment are desirable since the down-converted photons are emitted in a 3° cone, with correlated signal and idler photons positioned exactly opposite each other in the cone (see Fig. 3). Thus, the relative position in the three-dimensional cone is the important alignment consideration in detector alignment, and not just that both detectors detect some down-converted light at some location in the cone.

Fine alignment of the detectors can also be achieved by accurately aligning the BBO crystal and detectors such that all three components are at the same height off the optical table. Down-converted light can be selected such that only correlated pairs are measured by suitably placing two diaphragms downstream of the BBO crystal.

![Diagram of BBO Crystal and Down-Converted Light Cone](image)

**FIG. 3.** Down-converted light cone.

As the main laser beam and crystal alignment were completed, and all the necessary software and experimental protocol have been set up and written, after the above improvements
and corrections to the current experiment have been made, the lab should be able to serve as a photodetector metrology lab for Fermilab. Specifically, the setup will allow for the independent verification of the photodetection efficiencies of SiPMs that Fermilab purchases from outside vendors.

V. CONCLUSION

An optical lab that uses spontaneous parametric down-conversion to calibrate silicon photomultipliers was set up, and the necessary experimental protocol and data acquisition software was readied and written. As a test of the protocol and software, two single pixel silicon photomultipliers were aligned; their dark count rates were recorded, revealing that laser fluorescence is one of the principle sources of background counts with the current setup. Before accurate calibration experiments can be done with the current setup, translation stages with 3º of freedom need to be installed to allow for accurate fine alignment of the detector placement, and laser fluorescence needs to be reduced through the use of better shielding. After those issues have been resolved, the lab can serve as an efficient accurate metrology lab to calibrate photodetectors used at Fermilab.

VI. ACKNOWLEDGEMENTS

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APPENDIX

Assuming that the probability of observing \( n \) background events (either dark pulses or stray photons) in a detector \( D \) in a time \( \tau \) will then equal to:

\[
P_D(n \mid \tau) = \frac{(\lambda_D \tau)^n}{n!} \exp[-\lambda_D \tau]
\]  

(A1)

Where \( \lambda_D \) is the rate of random background events registered by the detector \( D \). The expectation value for the number of background events for the detector in a time \( \tau \) is then:

\[
< n > = \exp[-\lambda_D \tau] \sum_{n=0}^{\infty} \frac{(\lambda_D \tau)^n}{n!} = \lambda_D \tau
\]  

(A2)

In a given calibration run which lasts for a time \( \tau \) where \( N_m \) events are measured by detector \( D \), the average number of events that are caused by down-converted photons \( N' \) (ignoring the possibility of afterpulses) is then expected to be:

\[
N' \approx N_m - \exp[-\lambda_D \tau] \sum_{n=0}^{N_m} \frac{(\lambda_D \tau)^n}{n!}
\]  

(A3)

From Taylor’s theorem for polynomial expansions, the relative error of approximating the summed term in (A3) is:

\[
| R(\lambda_D \tau) | \leq \frac{\exp[\lambda_D \tau](\lambda_D \tau)^{N_m+1}}{(N_m + 1)!}
\]  

(A4)

Assuming that (A4) is much smaller than one (i.e., that \( N_m \) is very large), then (A3) can be approximated as:

\[
N' \approx N_m - \lambda_D \tau
\]  

(A5)

Where now the expected number of background terms depends linearly on the calibration time \( \tau \). As the standard deviation divided by the mean of the Poisson distribution is directly proportional to \((1/\sqrt{\lambda_D \tau})\) [9], and since the background counts \( \lambda_D \tau \) were generally very large (over several
hundred thousand background events were measured in preliminary tests with single-pixel SiPMs with the laser turned on but the down-converted beams blocked, (A5) can then be assumed to accurately calculate the true number of down-converted photons that triggered an event on detector $D$ in a typical calibration run. For the trigger detector then, $\lambda_D = B_t$ and for the accidentals coincidence rates, $\lambda_D = B_c$.

Assuming that the measured counts in the calibration experiment also followed a Poisson distribution, and that all the count rates were independent parameters, the variance of the calculated efficiency from equation (3.b) should then be:

$$\begin{align*}
(\Delta \eta)^2 &= \left(\frac{\partial \eta}{\partial (B_c \tau'_r)}\right)^2 (\Delta (B_c \tau'_r))^2 + \left(\frac{\partial \eta}{\partial (B_i \tau_r)}\right)^2 (\Delta (B_i \tau_r))^2 \\
&\quad+ \left(\frac{\partial \eta}{\partial S_a}\right)^2 (\Delta S_a)^2 + \left(\frac{\partial \eta}{\partial \rho}\right)^2 (\Delta \rho)^2
\end{align*}$$

(A6)

Where $B_c \tau'_r$ is the accidental coincidence counts, $B_i \tau_r$ is the background trigger single counts, $S_a$ is the number of counts measured by the trigger but were not accompanied by a coincident count in the conjugate detector, and $\rho$ is the efficiency of the filter in front of the conjugate detector.

Evaluating the partial derivatives and noting that the square root of the mean of a Poisson distribution is the standard deviation from the mean gives:

$$\begin{align*}
(\Delta \eta)^2 &= \left(\frac{1}{\rho(S_c + S_a - B_i \tau_r)}\right)^2 (B_c \tau'_r)^2 + \left(\frac{N_c - B_c \tau'_r}{\rho(S_c + S_a - B_i \tau_r)^2}\right)^2 (B_i \tau_r) \\
&\quad+ \left(\frac{N_c - B_c \tau'_r}{\rho(S_c + S_a - B_i \tau_r)^2}\right)^2 (S_a) + \left(\frac{N_c - B_c \tau'_r}{\rho^2(S_c + S_a - B_i \tau_r)}\right)^2 (\Delta \rho)^2
\end{align*}$$

(A7)

Which is formula (4).
REFERENCES


