CDF Top Quark Mass Measurement with the Matrix Element Method

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Outline

• Top mass measurements
• Channel and Event Selection
• Matrix element method
• Study of the integration methods: pMC, qMC
• Analysis of the preliminary results with the new TF
• Study of the sensitivity to the $\Delta JES$
• Future development of the analysis
Tevatron Collider

- Proton - Antiproton collider
- Completed in 1983
- Main achievement: Top quark discovery, 1995
- Energy reached Run II: $\sqrt{s} = 1.96 \, TeV$

Lead in top physics:
- Production properties (cross sections, $A_{FB}$)
- Decay properties (width, BR)
- Intrinsic properties (mass, spin, charge)
- Exotic searches involving top quarks
Top Mass Measurement: Previous results

World comb. 2014 : 0.44% Precision
Top Mass measurements: Advances in Precision

• D0 final measurement in lepton+jets:

\[
m_t = 174.98 \pm 0.58_{stat+JES} \pm 0.49_{syst} \ \text{GeV/c}^2 = 174.98 \pm 0.76 \ \text{GeV/c}^2
\]

\((0, 43\% \text{ precision})\)

PRL 113, 032002 (2014); PRD 91, 112003 (2015)

• CMS 7 + 8 TeV measurements in all channels: (Latest)

\[
m_t = 172.44 \pm 0.13_{stat} \pm 0.44_{syst} \ \text{GeV/c}^2 = 172.44 \pm 0.48 \ \text{GeV/c}^2
\]

\((0.28\% \text{ precision!})\)

PRD 93, 072004 (2016)

• CDF latest measurement aim:
  • Reach the highest possible precision From CDF Data.
  • Examine tension between LHC and Tevatron Results
CDF Top Mass Measurement: Channels

• Top decay Branching Ratio: \( t \rightarrow Wb \sim 100\% \)

\( tt \) decay signatures:
• \textit{Dilepton} events: both W bosons decay into an \( e\nu \) or \( \mu\nu \) final state.
  - Lowest branching ratio: \( \sim 7\% \) (including \( \tau \) leptons)
  - Two undetected neutrinos: Unconstrained kinematics

• \textit{Hadronic} events: both W bosons decay hadronically (6 jets)
  - Highest branching ratio: \( \sim 55\% \)
  - Large QCD multi-jet background

• Lepton + jets events: one W boson decay hadronically, the other into an \( e\nu \) or \( \mu\nu \).
  - Characterised by an isolated lepton, four jets, missing transverse energy.
  - Branching ratio: \( \sim 38\% \) (\( \tau \) events included)

\( \tau \) events: if \( \tau \) decays into \( e \) or \( \mu \), it appears in the electron/muon+jets signal sample.
CDF Top Mass Meas.: L+jets event selection / Background

- Lepton + jets: event signature:
  - High transverse momentum $p_T$ charged lepton;
  - Large missing transverse energy $\not{E}_T$ (escaping neutrino from $W$ decay in the final state);
  - At least 4 jets;

- Tight or loose jets:
  - Tight jet: $E_T > 20\text{GeV}$, $|\eta| \leq 2.0$
  - Loose jet: $E_T > 12\text{GeV}$, $|\eta| \leq 2.4$

- 5 subsamples based on the number of identified (tagged) b-jets (T or L):
  - 0-tag, 1-tagL, 1-tagT, 2-tagL, 2-tagT.

- Background: non- $t\bar{t}$ events that mimics the L+jets signature:
  - $W$+jets ($W+\bar{b}b$, $W+c\bar{c}$, $W+c$, $W+LF$) Included in the Likelihood
  - QCD ("fake" electrons, secondary electron): reduced by selection cuts.
  - other: (Single-top, Diboson ($WW,WZ,ZZ$), $Z$+jets)
CDF Latest measurement: Improvement respect to past analysis

• Increase of Integrated Luminosity: Exploiting the full CDF Run II Dataset.
  • from $5.6 \, fb^{-1}$ to $9.0 \, fb^{-1}$: $\sim 60\%$ more data;

• Inclusion of new sample categories:
  • untagged category: 0-tag;
  • loose categories: 1-tagL, 2tagL;

• For the first time in CDF analysis the Background Matrix Element modelling of the likelihood is included;

• Inclusion and refinement of the quasi-MC method in the Integration code;

• Smaller systematic uncertainties on the final measurement by introducing several new signal and background modelling;

• NLO signal MC: Reduction of uncertainty in Calibration Procedure
Matrix Element Technique

• Full kinematic and topological information in any event is considered.
• Calculation of the Matrix element to find the probability for the event:
  • Integration over phase space \( d\Phi(x) \ [t\bar{t} \rightarrow b(\nu)b(qq')] \)
  • 32 variables of integration: \( \text{constraints} \)
  • 19 variables of integration:

• Signal and Background:

\[
L_{ev}(y|m_t, \Delta JES) = a(f_{\text{sig}})L_{\text{sig}}(y|m_t, \Delta JES) + b(f_{\text{bkg}})L_{\text{bkg}}(y|\Delta JES)
\]

• Likelihood:

\[
L_{tot}(y|m_t, \Delta JES) = \prod_{i=1}^{N} L_{ev}(y|m_t, \Delta JES)
\]

• Calibration of Method:

• Pseudo-experiment to correct biases and missing background modelling
• A fast and reliable integration algorithm is essential
Validation of the integration method: Pull Distribution pMC

- pMC integration: Random sequence of points with importance sampling;

- Relative error behaviour: \( \frac{\Delta I}{I} \sim O(N^{-1/2}) \)

- Pull distribution:
  
  Meaning:
  \[
  \langle W_{k,ij} \rangle = \frac{1}{N} \sum_{l=1}^{N} W_{k,ij,l}
  \]
  
  Standard dev:
  \[
  \sigma_{k,ij} = \sqrt{\frac{1}{N-1} \sum_{l=1}^{N} (W_{k,ij,l} - \langle W_{k,ij} \rangle)^2}
  \]
  
  Pull variable:
  \[
  \delta_{k,ij,l} = \frac{W_{k,ij,l} - \langle W_{k,ij} \rangle}{\sigma_{k,ij}}
  \]

  \(k = \text{event}\)
  
  \(i, j = (\Delta_{JES}, M_t) \text{ bins}\)
  
  \(l = \text{integration}\)
Validation of the integration method: Pull Distribution pMC

- Problem in the pull distribution: 2 events gave a spike

\[ \delta_{k,i,j,l} \quad \forall (k, i, j, l) \quad (l \in [1, 22]) \]
Validation of the integration method: Pull Distribution pMC

- 22 integrations with random seed of ~1000 events

\( (MC, \ m_t = 173 \ GeV/c^2, \ \Delta J_{ES} = 0 \ \sigma, \ 1\text{TagT}) \)

\( \delta_{k,i,j,l} \quad \forall (k, i, j, l) \ \{l \in [1, 22] \land k \neq (58, 330)\} \)
Validation of the integration method: Pull Distribution qMC

- qMC integration: sobol sequence of points instead of random sequence;
- Sobol sequence: LDS (Low-discrepancy sequence)
  (uniformly spread across the integration domain)
- Expected relative error: \( \Delta I / I \sim \mathcal{O}(N^{-1+\varepsilon}) \)
- Introduction of random scrambling:
  Owen + Faure-Tezuka: random scrambling preserving LDS
- Importance sampling embedded in the code;
- Expected faster convergence of the integral.
Validation of the integration method : Pull Distribution qMC

- Problem in the pull distribution: the same problematic events.

\[ \delta_{k,ij,l} \forall (k, i, j, l) \ (l \in [1, 22]) \]
Validation of the integration method: Pull Distribution qMC

• 22 integrations with random seed of ~1000 events

\[(MC, m_t = 173 \text{ GeV}/c^2, \Delta_{JES} = 0 \text{ \sigma}, 1\text{Tag}\text{T})\]

\[
\delta_{k,ij,l} \forall (k, i, j, l) \ \{l \in [1, 22] \land k \neq (58, 330)\}
\]
q-MC vs p-MC: estimation of precision

Given same termination parameters: (time, max n points, precision)

Compare the histograms of:

\[ r_{k,ij} = \frac{\sigma_{k,ij}}{<W_{k,ij}>} \quad \forall \ k, i, j \]

More quantitative testing needed: time and convergence.
New Transfer Functions

- Transfer Functions. Probability density $T(x|y, \Delta_{JES})$ relating:
  - measured quantities $\eta_{jet}, \phi_{jet}, p_{t,jet}$
  - parton-level quantities $\eta_{part}, \phi_{part}, p_{t,part}$
  (observed in detectors) (used for ME calculation)

- TF can be factorised in:
  $$T(x|y, \Delta_{JES}) = T_a(\eta_{jet}, \phi_{jet}|\eta_{part}, \phi_{part})T_m(p_{t,jet}|p_{t,part})$$

- New TF derived from MC simulations including loose categories ($E_t > 12 GeV$):
  - 1TagL
  - 2TagL

- For event with only tight categories old/new should produce the same results
Comparison between New/Old TF: Single event

MC Generated: PYTHIA, $M_t = 170 GeV/c^2$, $\Delta JES = 0\sigma$, 0Tag category.

Old TF

<table>
<thead>
<tr>
<th>Event</th>
<th>Ntight</th>
<th>NLoose</th>
<th>nbtags</th>
<th>missingEt</th>
<th>ev.lpt</th>
<th>jet1pt</th>
<th>jet2pt</th>
<th>jet3pt</th>
<th>jet4pt</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td>49.4882</td>
<td>52.4504</td>
<td>64.0641</td>
<td>70.1314</td>
<td>40.4148</td>
<td>28.5606</td>
</tr>
</tbody>
</table>
Comparison between New/Old TF

1000 ev MC Generated: PYTHIA, $M_t = 170 GeV/c^2$, $\Delta JES = 0 \sigma$, 1 Tag T.

Sample  
Top Mass 173 GeV/c^2  
$\Delta JES$ 0 $\sigma$  
Results  
$M_t(2DPeak)$ 172.5 GeV  
$\Delta JES(2DPeak)$ $-1\sigma$

Sample  
Top Mass 173 GeV/c^2  
$\Delta JES$ 0 $\sigma$  
Results  
$M_t(2DPeak)$ 178.5 GeV  
$\Delta JES(2DPeak)$ 2 $\sigma$
Study of the sensitivity on Djes

- Analysis of 3 MC samples with different parameters:
  - signal events, $M_t = 172.5 GeV/c^2$, $\Delta_{JES,MC} = \{-1, 0, +1\}$, 1TagT & 2TagT categories;
  - 10000 events for every sample.

- Calculate total $log(L)$:
  $$log(L)_{ij} = \sum_{k=1}^{10000} ln(W_{k,ij});$$

- Create 1D histogram of profiled likelihood: $logL(M_t), logL(\Delta_{JES})$;
  (using the profiled likelihood method)

- Extract $M_t, \Delta_{JES}$ (assuming gaussian behaviour of the likelihood in the limit of large statistics);

- Plot the dependency $M_t, \Delta_{JES}$ to the input $\Delta_{JES,MC}$;

- Expected linear dependency to be corrected with calibration.
Study of the sensitivity on Djes

\[ \Delta_{JES,MC} = -1 \sigma \]

<table>
<thead>
<tr>
<th>Sample</th>
<th>ttop25_mc_jes-1_btag_jpt20_hs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{t,MC} )</td>
<td>172.5 GeV/c^2</td>
</tr>
<tr>
<td>( \Delta_{JES,MC} )</td>
<td>-1 \sigma</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_t(\text{fit}) )</td>
<td>172.3 ± 0.2 GeV/c^2</td>
</tr>
<tr>
<td>( \Delta_{JES(\text{fit})} )</td>
<td>-1.74 ± 0.04 \sigma</td>
</tr>
</tbody>
</table>
Study of the sensitivity on Djes

\[ \Delta_{JES,MC} = 0 \sigma \]

<table>
<thead>
<tr>
<th>Sample</th>
<th>ttop25_mc_jes0_btag_jpt20_hs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_t,MC )</td>
<td>172.5 GeV/c^2</td>
</tr>
<tr>
<td>( \Delta_{JES,MC} )</td>
<td>0 \sigma</td>
</tr>
<tr>
<td>Results</td>
<td></td>
</tr>
<tr>
<td>( M_t(\text{fit}) )</td>
<td>170.9 \pm 0.2 GeV/c^2</td>
</tr>
<tr>
<td>( \Delta_{JES}(\text{fit}) )</td>
<td>-1.32 \pm 0.05 \sigma</td>
</tr>
</tbody>
</table>

LogL

LogL

LogL

LogL

LogL

LogL

LogL

LogL

LogL

LogL
Study of the sensitivity on Djes

\[ \Delta_{JES,MC} = 1 \sigma \]

<table>
<thead>
<tr>
<th>Sample</th>
<th>ttop25_mc_jes+1.btag_jpt20_hs</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_{t,MC} )</td>
<td>172.5 GeV/c^2</td>
</tr>
<tr>
<td>( \Delta_{JES,MC} )</td>
<td>1 ( \sigma )</td>
</tr>
<tr>
<td>Results</td>
<td></td>
</tr>
<tr>
<td>( M_t(\text{fit}) )</td>
<td>169.8 \pm 0.2 GeV/c^2</td>
</tr>
<tr>
<td>( \Delta_{JES}(\text{fit}) )</td>
<td>-0.90 \pm 0.05 ( \sigma )</td>
</tr>
</tbody>
</table>
Study of the sensitivity on Djes

<table>
<thead>
<tr>
<th>$\Delta JES_{MC}$</th>
<th>$-1\sigma$</th>
<th>$0\sigma$</th>
<th>$1\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_t (fit)$</td>
<td>$172.3 \pm 0.2 , \text{GeV/c}^2$</td>
<td>$170.9 \pm 0.2 , \text{GeV/c}^2$</td>
<td>$169.8 \pm 0.2 , \text{GeV/c}^2$</td>
</tr>
<tr>
<td>$\Delta JES (fit)$</td>
<td>$-1.74 \pm 0.04 , \sigma$</td>
<td>$-1.32 \pm 0.05 , \sigma$</td>
<td>$-0.90 \pm 0.05 , \sigma$</td>
</tr>
</tbody>
</table>

Linear dependence: expected to be corrected with calibration!
Future development of the analysis

- Further understand of qMC integration and implement error estimation;
- Include total cross section and acceptance as normalisation of the weights;
- Study the sensitivity on $\Delta_{JES}$ and $M_t$ with the complete weight definition;
- Debug the new TF;
- Test the methods for loose categories;
- Combine signal and background likelihood;
- Final calibration (pseudo-experiments);
Matrix Element Method: Motivation

• Provides superior Statistical sensitivity in the extraction of SM parameters;

• Completeness of information exploited in each event:
  • The superior sensitivity is achieved by taking into account the full topological and kinematic information in a given event;

• Can be used to determine several parameters:
  • Theoretical parameters describing the physics of the processes measured;
  • Experimental parameters: describing the detector response;

• Theoretical assumption about the process under study (PDF, ME, TF) are used in the most efficient manner:
  • In the limit which all the event probabilities are known, by the Neyman-Pearson Lemma, the likelihood is an optimal test statistic.
Validation of the integration method: Pull Distribution qMC

- qMC integration: sobol sequence of points instead of random sequence;

- Sobol sequence: LDS (Low-discrepancy sequence)

\[ D^*_N(P_N) = \sup_{\alpha \in A} \left| \frac{\#\{P_N \in A\}}{N} - \lambda(A) \right|, \quad A = \prod_{i=1}^{d} [0, b_i) \ \forall b_i \ \text{s.t.} \ 0 \leq b_i < 1 \]

- Koksma–Hlawka inequality:

\[ \left| \frac{1}{N} \sum_{i=1}^{N} f(x_i) - \int_{I} f(u) du \right| \leq V(f) D^*_N(x_1, \ldots, x_n) \]

- Expected relative error: \( \Delta I/I \sim O(N^{-1+\varepsilon}) \) (faster convergence)

- Owen + Faure-Tezuka scrambling preserving LDS

- Importance sapling embedded in the code.
Integration Framework: Fermigrid

Local Machine -> Director -> Master -> Libraries, paths ...

FermiGrid

Workers: 1 2 3 4 5 6 ...

- Program installation;
- Learn job submission procedure;
- Learn program errors handling;
- Learn to analyse data format consistent with ME analysis code
Signal and Background: Monte Carlo Samples / Validation

• Signal:
  - $t\bar{t}$ signal: Powheg + Phytia \quad S.Frixione \textit{et al.}, JHEP07 (2007)

• Background:
  - Diboson: Pythia 6 \quad T. Sjöstrand \textit{et al.}, JHEP06, 026 (2006)
  - Single top: Madgraph 4 + Pythia \quad J. Alwall \textit{et al.}, JHEP09, 028 (2007)
  - QCD: Data with lepton failing one of the “good lepton” criteria

• Validation of Samples:
  - Validation plots: C. Tosciri-Laurea-University of Pisa
  - Compare quantity of interest between data and MC events of the samples
Signal and Background: MC samples - Validation Plots

CDF Preliminary

**Missing E 1tagT**
- Data
- tt
- W+jets
- QCD
- SingleTop
- Diboson
- Z+jets

\[ \int L \ dt = 9.0 \ fb^{-1} \]

CDF Preliminary

**P_{\text{lepton}} 1tagL**
- Data
- tt
- W+jets
- QCD
- SingleTop
- Diboson
- Z+jets

\[ \int L \ dt = 9.0 \ fb^{-1} \]

CDF Preliminary

**P_{\text{lepton}} b-jets 1tagL**
- Data
- tt
- W+jets
- QCD
- SingleTop
- Diboson
- Z+jets

\[ \int L \ dt = 9.0 \ fb^{-1} \]

CDF Preliminary

**\eta_{\text{lepton}} 2tagL**
- Data
- tt
- W+jets
- QCD
- SingleTop
- Diboson
- Z+jets

\[ \int L \ dt = 9.0 \ fb^{-1} \]
### Backup Slides: Systematic Uncertainties

#### List of systematic uncertainties on $m_t$.

<table>
<thead>
<tr>
<th>Systematic Source</th>
<th>Uncertainty (GeV/$c^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration</td>
<td>0.10</td>
</tr>
<tr>
<td>Monte Carlo Generator</td>
<td>0.37</td>
</tr>
<tr>
<td>Initial State Radiation and Final State Radiation</td>
<td>0.15</td>
</tr>
<tr>
<td>Residual JES</td>
<td>0.49</td>
</tr>
<tr>
<td>$b$-JES</td>
<td>0.26</td>
</tr>
<tr>
<td>Lepton $p_T$</td>
<td>0.14</td>
</tr>
<tr>
<td>Multiple Hadron Interactions</td>
<td>0.10</td>
</tr>
<tr>
<td>PDFs</td>
<td>0.14</td>
</tr>
<tr>
<td>Background Modeling</td>
<td>0.33</td>
</tr>
<tr>
<td>Color Reconnection</td>
<td>0.37</td>
</tr>
<tr>
<td>Total</td>
<td>0.88</td>
</tr>
</tbody>
</table>

- Remove overlap
- $Bkg \text{ in } L_{ev}$
- New sgn MC
## Expected and Observed Sample composition

<table>
<thead>
<tr>
<th></th>
<th>0-tag</th>
<th>1-tagL</th>
<th>1-tagT</th>
<th>2-tagL</th>
<th>2-tagT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wbbbar</td>
<td>125.6</td>
<td>177.1</td>
<td>82.2</td>
<td>27.3</td>
<td>17.0</td>
</tr>
<tr>
<td>Wccbar</td>
<td>384.8</td>
<td>112.9</td>
<td>52.5</td>
<td>4.1</td>
<td>2.6</td>
</tr>
<tr>
<td>Wc</td>
<td>186.7</td>
<td>66.8</td>
<td>25.9</td>
<td>2.3</td>
<td>1.3</td>
</tr>
<tr>
<td>W+light j</td>
<td>1580.9</td>
<td>170.7</td>
<td>77.2</td>
<td>2.9</td>
<td>1.9</td>
</tr>
<tr>
<td>Z+jets</td>
<td>169.4</td>
<td>25.2</td>
<td>13.7</td>
<td>2.0</td>
<td>1.3</td>
</tr>
<tr>
<td>single top</td>
<td>13.9</td>
<td>16.5</td>
<td>8.2</td>
<td>6.7</td>
<td>4.6</td>
</tr>
<tr>
<td>Diboson</td>
<td>166.3</td>
<td>31.0</td>
<td>17.9</td>
<td>2.7</td>
<td>1.8</td>
</tr>
<tr>
<td>QCD</td>
<td>623.2</td>
<td>119.9</td>
<td>60.3</td>
<td>60.3</td>
<td>6.3</td>
</tr>
<tr>
<td><strong>Total Bkg</strong></td>
<td>3250.8</td>
<td>720.1</td>
<td>337.9</td>
<td>49.0</td>
<td>36.8</td>
</tr>
<tr>
<td><strong>ttbar</strong></td>
<td>959.7</td>
<td>998.6</td>
<td>1086.3</td>
<td>331.3</td>
<td>425.5</td>
</tr>
<tr>
<td><strong>Expected total</strong></td>
<td>4210.5</td>
<td>1718.7</td>
<td>1424.2</td>
<td>380.3</td>
<td>462.3</td>
</tr>
<tr>
<td><strong>Observed</strong></td>
<td>4474</td>
<td>1711</td>
<td>1434</td>
<td>365</td>
<td>375</td>
</tr>
</tbody>
</table>
### Selection requirements for event-category

<table>
<thead>
<tr>
<th></th>
<th>0-tag</th>
<th>1-tagT</th>
<th>1-tagL</th>
<th>2-tagT</th>
<th>2-tagL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton $E_T$ [GeV]</td>
<td>&gt; 20</td>
<td>&gt; 20</td>
<td>&gt; 20</td>
<td>&gt; 20</td>
<td>&gt; 20</td>
</tr>
<tr>
<td>Lepton $</td>
<td>\eta</td>
<td>$</td>
<td>0 – 1</td>
<td>0 – 1</td>
<td>0 – 1</td>
</tr>
<tr>
<td>$\not{E}_T$ [GeV]</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Leading 3 jets $E_T$ [GeV]</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Leading 3 jets $</td>
<td>\eta</td>
<td>$</td>
<td>0 – 2</td>
<td>0 – 2</td>
<td>0 – 2</td>
</tr>
<tr>
<td>4$^{\text{th}}$ jet $E_T$ [GeV]</td>
<td>&gt; 20</td>
<td>&gt; 20</td>
<td>&gt; 12</td>
<td>&gt; 20</td>
<td>&gt; 12</td>
</tr>
<tr>
<td>4$^{\text{th}}$ jet $</td>
<td>\eta</td>
<td>$</td>
<td>0 – 2</td>
<td>0 – 2</td>
<td>0 – 2.4</td>
</tr>
<tr>
<td>Extra jets $E_T$ [GeV]</td>
<td>&lt; 20</td>
<td>Any loose or &gt; 1 tight</td>
<td>Any loose or &gt; 1 tight</td>
<td>Any loose or &gt; 1 tight</td>
<td>Any loose or &gt; 1 tight</td>
</tr>
</tbody>
</table>
Matrix Element Technique 2

- Determines $P_{ev}(y|a)$ from:
  - Vector of observed data: $y \longrightarrow$ event kinematics
  - Vector of model parameters: $a \longrightarrow m_t, \Delta JES$

- Probability: Integration over all phase space $
  d\Phi(x) \left[ t\bar{t} \rightarrow b(l\nu)b(qq') \right]$

- 32 variables of integration:
  - 2 initial particles (8 var.)
  - 6 final particles (24 var.)
- 19 variables of integration:
  - $M_{t,lep}^2, M_{t,had}^2, M_{W,lep}^2, M_{W,had}^2$
  - $\beta = \ln \frac{p_q}{p_{q'}}$, $p_T(t\bar{t})$, $m_{1...4}$, $\eta_{1...4}$, $\phi_{1...4}$

- Signal and Background

  $P_{ev}(y|m_t, \Delta JES) = A(y) \left[ f P_{sig}(y|m_t, \Delta JES) + (1 - f) P_{W+jets}(y|\Delta JES) \right]$

- Likelihood:

  $L(y|m_t, \Delta JES) = \prod_{i=1}^{N} P_{ev}(y|m_t, \Delta JES)$
Comparison between New/Old TF

100 ev MC Generated: PYTHIA, $M_t = 170 GeV/c^2$, $\Delta_{JES} = 0\sigma$, 0Tag.

![LogL vs. M vs. $\Delta_{JES}$ for New TF](image1)

![LogL vs. M vs. $\Delta_{JES}$ for Old TF](image2)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Top Mass</th>
<th>$M_t(2DPeak)$</th>
<th>$\Delta_{JES}(2DPeak)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>$170 GeV/c^2$</td>
<td>$162.5 GeV$</td>
<td>$-2\sigma$</td>
</tr>
<tr>
<td>Old</td>
<td>$170 GeV/c^2$</td>
<td>$172.5 GeV$</td>
<td>$2.6\sigma$</td>
</tr>
</tbody>
</table>